MA135 / MA160 Algebra

Consider the following text:

> If school is cancelled, I will go into town with my friends.
> School is not cancelled.
> Therefore, I will not go into town with my friends.

This is an example of an invalid argument – more later.

Recap of modular arithmetic

$$7 + 8 \equiv 3 \mod 12$$
$$7 - 8 = -1 \equiv 11 \mod 12$$
$$7 \cdot 8 \equiv 8 \mod 12$$
$$7^{-1} = ? \mod 12$$
$$= 7 \quad \text{by Euclid's algorithm (or trial and error).}$$

We are interested in mod 2 arithmetic in the context of LOGIC.

Consider now the function (called a Boolean function)

$$f(x,y) = xy + x + y \mod 2$$

Let's list all possibilities in a table:

x	y	$xy+x+y \bmod 2$
1	1	$1(1)+1+1=1$
1	0	$1(0)+1+0=1$
0	1	$0(1)+0+1=1$
0	0	$0(0)+0+0=0$

How does this have a connection with logic?

Consider the following statements:

P_1 = "A square has 4 sides" (T)

P_2 = "A triangle has 4 sides" (F)

Q_1 = "Edinburgh is the capital of Scotland" (T)

Q_2 = "Edinburgh is the capital of Gt. Britain" (F)

Next consider combinations of these using the connective 'or':

Compound statement

P_1 or Q_1: "A square has 4 sides or a triangle has 4 sides" (True)

P_1 or Q_2: "A square has 4 sides or Edinburgh is the capital of Gt. Britain" (True)

P_2 or Q_1: "A triangle has 4 sides or Edinburgh is the capital of Scotland" (True)

P_2 or Q_2: "A triangle has 4 sides or Edinburgh is the capital of Gt. Britain" (False)

The meaning of 'OR' is described by the following table:

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

Recall (mod 2)

Compare and contrast

x	y	$f(x,y) = xy+x+y \bmod 2$
1	1	1
1	0	1
0	1	1
0	0	0

called a Boolean function

We take the above table as the definition of OR; the table is called a truth table.

How about P AND Q?

Eg: "A square has 4 sides AND Edinburgh is the capital of Gt. Britain". (False)

This is P_1 AND Q_2.

"A square has 4 sides AND Edinburgh is the capital of Scotland." (True).

(P_1 AND Q_1)

The connective AND is defined by the following truth table:

P	Q	P AND Q
T	T	T
T	F	F
F	T	F
F	F	F

Boolean function

$f(x,y) = xy \pmod 2$

x	y	
1	1	1
1	0	0
0	1	0
0	0	0