Aim: State Leray's nerve theorem as motivation for studying the clique simplicial complex K_Σ.

Let $\mathcal{U} = \{U_1, U_2, \dots, U_n\}$ be a collection of sets. Then **nerve** $N\mathcal{U}$ is a simplicial complex with

$$V = \{U_1, U_2, \dots, U_n\}$$

with a d-simplex

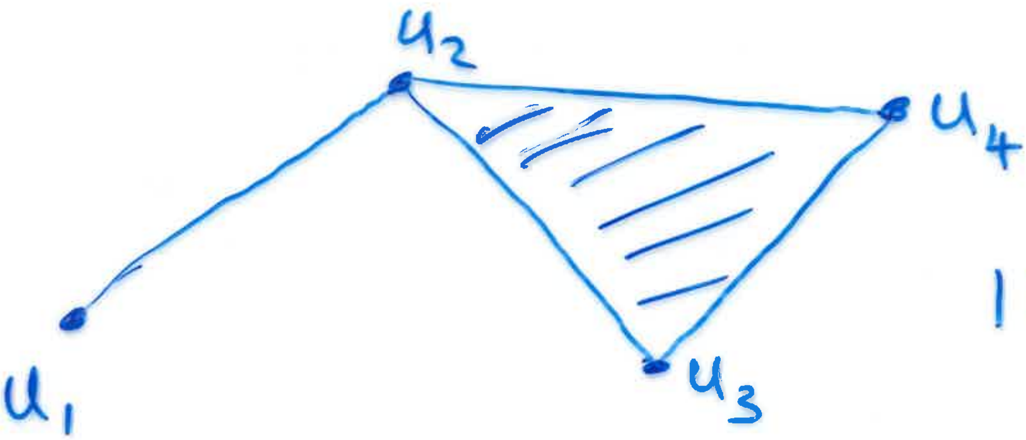
$$\sigma = \{U_{i_0}, U_{i_1}, \dots, U_{i_d}\}$$

whenever

$$U_{i_0} \cap U_{i_1} \cap \dots \cap U_{i_d} \neq \emptyset.$$

Example

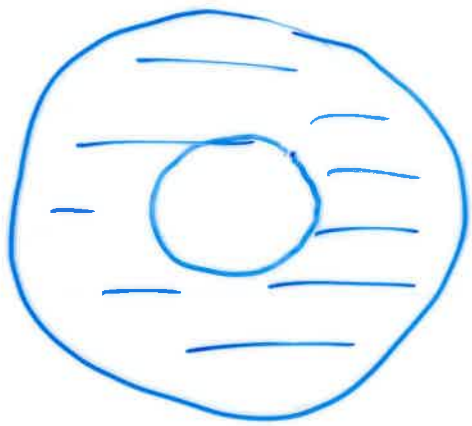
$$\mathcal{U} = \{U_1 = \{1,2,3\},\ U_2 = \{3,4,5\},\ U_3 = \{4,5,6\},\ U_4 = \{5,6,7\}\}.$$



$|N\mathcal{U}|$

<u>Example 2</u> Consider

$$X = \{ z \in \mathbb{C} : 1 \leq |z| \leq 2 \}$$

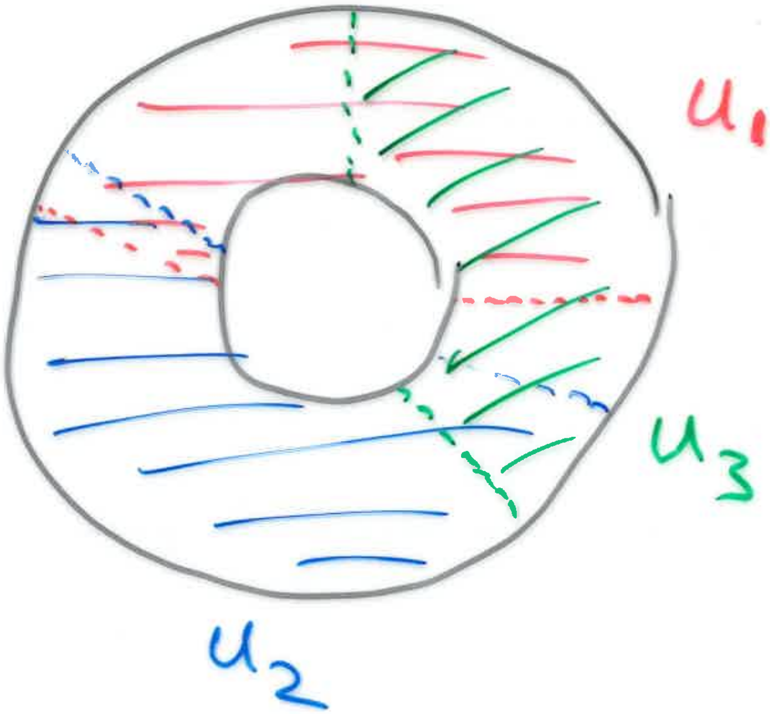
$X =$ 

Consider

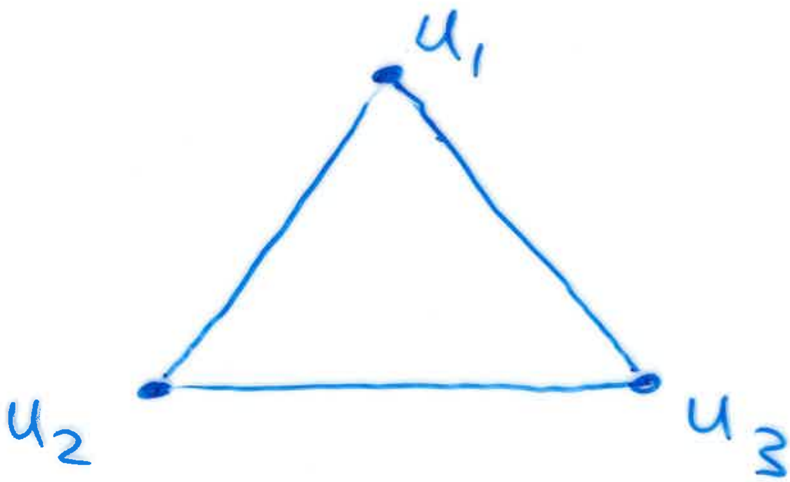
$$U_1 = \{ z \in X : 0 < \mathrm{Arg}(z) < \tfrac{5\pi}{6} \}$$

$$U_2 = \{ z \in X : \tfrac{4\pi}{6} < \mathrm{Arg}(z) < \tfrac{11\pi}{6} \}$$

$$U_3 = \{ z \in X : -\tfrac{2\pi}{6} < \mathrm{Arg}(z) < \tfrac{3\pi}{6} \}$$

$X =$ 

So $\mathcal{U} = \{U_1, U_2, U_3\}$ is an open cover of X. i.e. each U_i is open and $X = U_1 \cup U_2 \cup U_3$.

$|N\mathcal{U}| =$ 

Note that $|N\mathcal{U}|$ is homotopy equivalent to X.

Defn A space X is contractible if it is homotopy equivalent to a singleton space $\{1\}$.

Theorem (J. Leray) Let $\mathcal{U} = \{U_1, U_2, \ldots, U_n\}$ be an open cover of a compact space X such that every non-empty intersection of finitely many sets in $\mathcal{U}$ is contractible. Then X is homotopy equivalent to the nerve $|N\mathcal{U}|$.