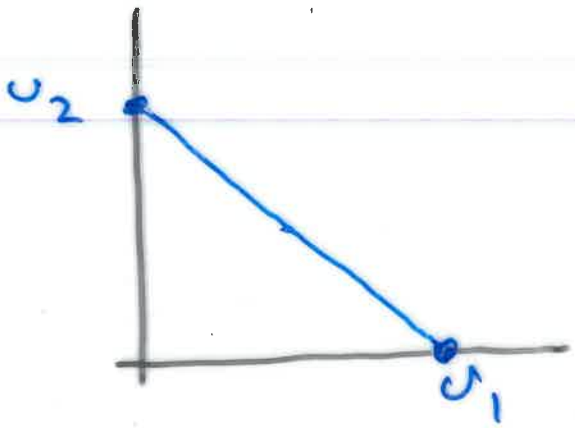
<u>Defn</u> Given a set $S = \{v_1, \dots, v_k\} \in \mathbb{R}^n$ of vectors in $\mathbb{R}^n$, we define the <u>convex</u> <u>hull</u>

$$\mathrm{Conv}(S) = \left\{ w = \lambda_1 v_1 + \cdots + \lambda_k v_k \ : \ \lambda_i \geq 0 \text{ all } i \text{ and } \sum_{i=1}^{k} \lambda_i = 1 \right\}$$

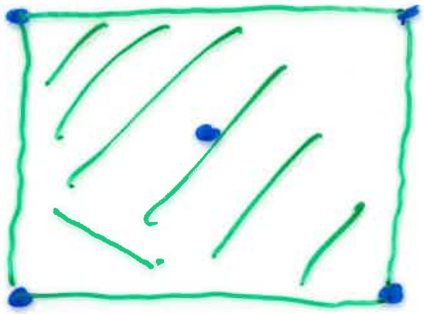
<u>Example</u> $S = \{v_1 = (1,0), v_2 = (0,1)\} \subseteq \mathbb{R}^2$

$$\mathrm{Conv}(S) = \{\lambda_1 v_1 + \lambda_2 v_2 \ : \ \lambda_1, \lambda_2 \geq 0, \ \lambda_1 + \lambda_2 = 1\}$$



<u>Example</u> $S = \{v_1 = (0,0), v_2 = (1,0), v_3 = (0,1)$
$v_4 = (1,1), v_5 = (\frac{1}{2}, \frac{1}{3})\}$

$\subseteq \mathbb{R}^2$.



Conv(S) is a square region in $\mathbb{R}^2$.

Defn Let $v_0, v_1, \dots, v_k \in \mathbb{R}^d$ be $k+1$ vectors such that the k vectors

$$v_1 - v_0,\ v_2 - v_0,\ \dots,\ v_k - v_0$$

are linearly independent. We say

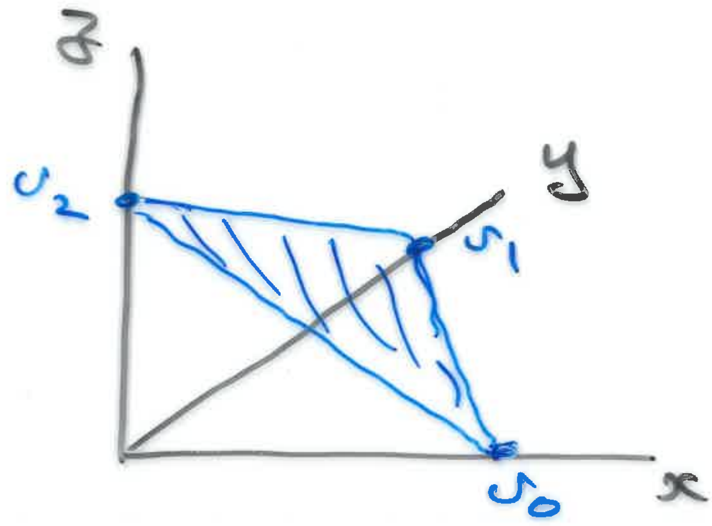
$$\mathrm{Conv}(\{v_0, v_1, \dots, v_k\})$$

is a geometric k-simplex.

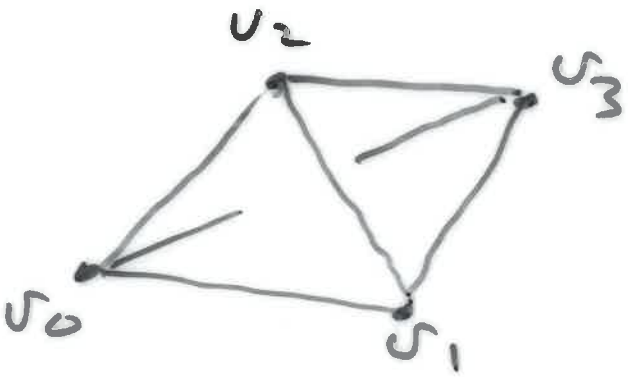
Example $S = \{ v_0 = (1,0,0)$, $v_1 = (0,1,0)$, $v_2 = (0,0,1) \} \subseteq \mathbb{R}^3$

$v_1 - v_0$, $v_2 - v_0$ are linearly independent.

So Conv(S) is a geometric 2-simplex.



Example Geometric 3-simplex



solid tetrahedron

Suppose (K, V) is a simplicial complex with finite vertex set $V = \{v_1, v_2, \ldots, v_n\}$ say.

Now identify v_i with the ith standard basis vector of $\mathbb{R}^n$,

position i,

$$v_i = (0, \ldots, 0, 1, 0, \ldots 0)$$

for each k-simplex

$$\sigma = \{v_{i_0}, v_{i_1}, \dots, v_{i_k}\}$$

we let

$$|\sigma| = \mathrm{Conv}(\{v_{i_0}, \dots, v_{i_k}\})$$

denote the corresponding geometric k-simplex.

<u>Defn</u> The geometric realization $|K|$ is the subset of $\mathbb{R}^n$ arising as the union of the geometric simplices $|\sigma|$ with $\sigma \in K$.

Note that $|K|$ is a topological subspace of $\mathbb{R}^n$.

Suppose given an $n \times n$ distance matrix $D = (d_{ij})$, recording dissimilarities between n items.

for any $\varepsilon > 0$ we can construct a simplicial complex (K_ε, V) with

$$V = \{1, 2, \dots, n\}$$

$$K_\varepsilon = \{\sigma \subseteq V : d_{ij} \leq \varepsilon \text{ for all } i,j \in \sigma\}$$

we can now associate the topological space $|K_\varepsilon|$.

<u>Example</u> 200 people were asked to visit Galway harbour at their convenience once during a 2-week period. They were asked to record the height of the water on their arrival, and then again 2 hours later, and then again 4 hours after their arrival. Each person returns their recordings $(h_0, h_2, h_4) \in \mathbb{R}^3$. From the set $S = \{x_i = (h_{i_0}, h_{i_2}, h_{i_4}) : 1 \le i \le 200\}$ a data analyst can construct

$$D = (d_{ij}) \quad \text{with}$$

$$d_{ij} = \| x_i - x_j \| \quad \text{Euclidean metric.}$$

The analyst could view the graph of the simplicial complex K_ε for various values of ε.

See computer demo.