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MA500 = CS4102 + CS4103

Geometric foundations of Data Analysis

Geometry: concerns distance & distance-preserving transformations

Statistics: largely concerns inferences about a population based on samples of the population.

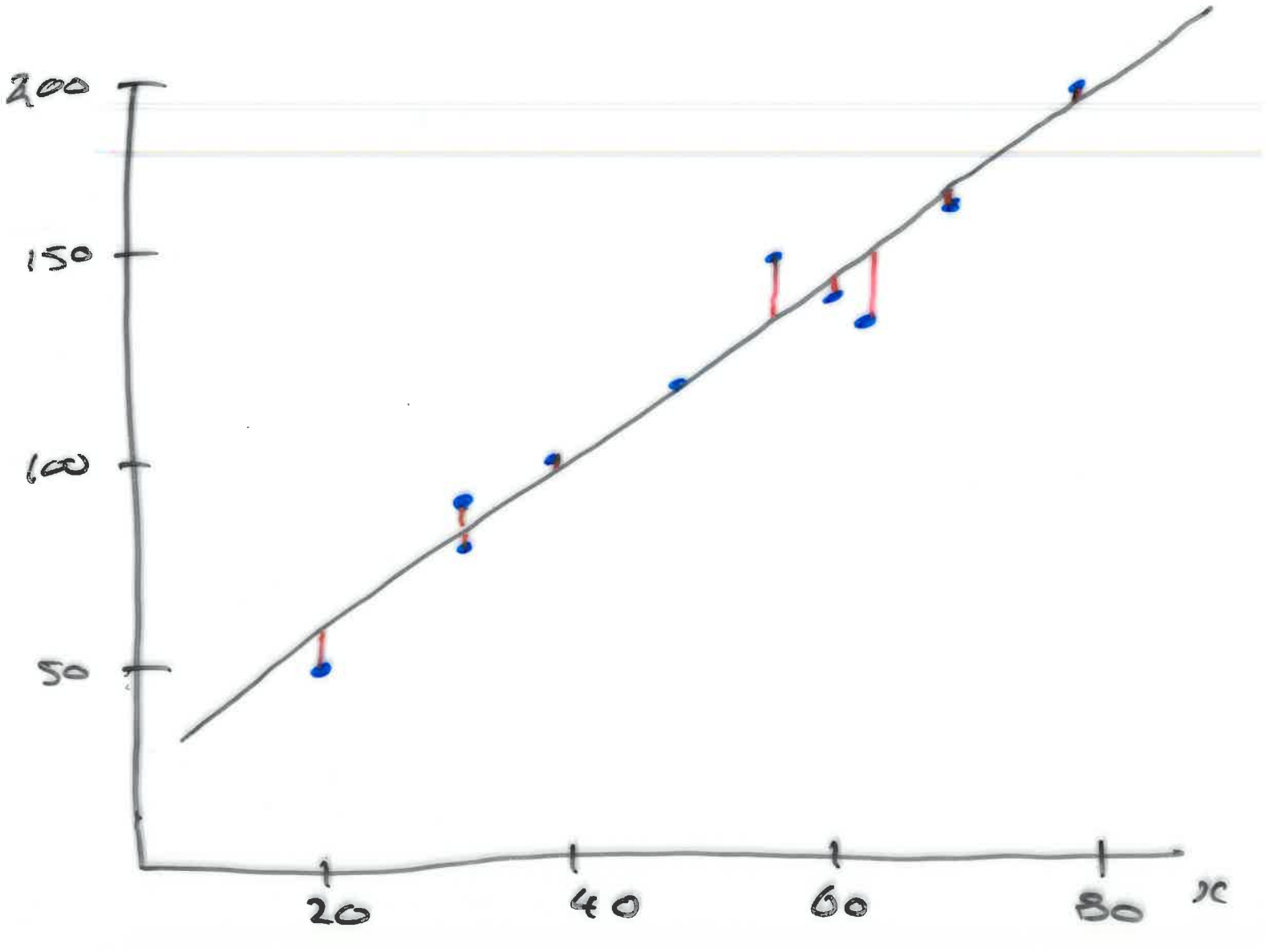
Probability: concerns inferences about a sample based on knowledge of the population

Data Analysis: Concerns the discovery and communication of meaningful patterns in data. Unlike statistics, it often deals with analyses where there is no assumed null-hypothesis. It often favours visualisation to communicate insight.

Least Squares Fitting

Consider a company that manufactures spare parts once per month in lots which vary in size according to demand.

Production run i	Lot size x_i	Person-hours y_i
1	30	73
2	20	50
3	60	128
4	80	170
5	40	87
6	50	108
7	60	135
8	30	69
9	70	148
10	60	132



insight into the relationship between lot size and person-hours can be gained by "fitting" a straight line to this data.

The fitted line is represented by

$$y = b_0 + b_1 x$$

where b_0, b_1 are chosen to be "best" in the following sense: they should minimise

$$Q = \sum_{i=1}^{n} (y_i - (b_0 + b_1 x_i))^2$$

where $n = 10$, x_i, y_i are given in the above table.

Here $Q = Q(b_0, b_1)$ is a function of b_0, b_1.

For a minimum we want

$$(*)\begin{cases} \dfrac{\partial Q}{\partial b_0} = -2\sum_{i=1}^{n}(y_i - (b_0 + b_1 x_i)) = 0 \\ \dfrac{\partial Q}{\partial b_1} = -2\sum_{i=1}^{n}(y_i - (b_0 + b_1 x_i))x_i = 0 \end{cases}$$

(*) are called the normal equations. They can be rewritten as

$$(*)\begin{cases} n b_0 + b_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \\ b_0 \sum_{i=1}^{n} x_i + b_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \end{cases}$$

two equations in two unknowns b_0, b_1.

The solution is :

$b_0 = 10.0$

$b_1 = 2.0$

and the fitted line is

$y = 10 + 2x$

So we "estimate" that the mean number of person-hours increases by two hours for each unit increase in lot size.

Matrix Notation

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \qquad X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \qquad B = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$$

(*) becomes

$$(*) \qquad X^t X B = X^t Y$$

Hence

$$B = (X^t X)^{-1} X^t Y$$