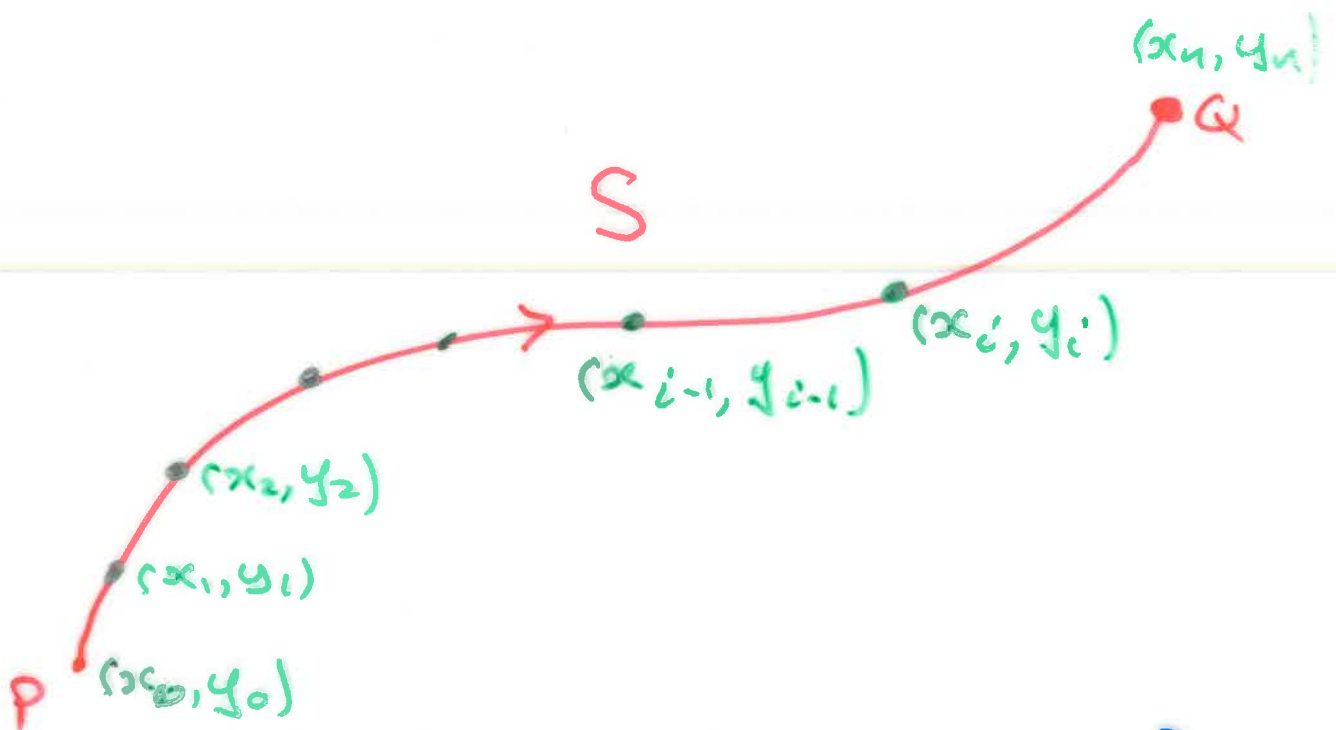


First test: Wednesday 9 October

More formally:

$$\int_S A(x,y) dx + B(x,y) dy =$$

$$\lim_{\|P\| \rightarrow 0} \sum A(x_i, y_i)(x_i - x_{i-1}) + B(x_i, y_i)(y_i - y_{i-1})$$



where $P = \{(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)\}$ is a sequence of points on S with (x_0, y_0) the initial point, and (x_n, y_n) the final point on S .

$$\|P\| = \max_{1 \leq i \leq n} \|(x_i, y_i) - (x_{i-1}, y_{i-1})\|$$

with

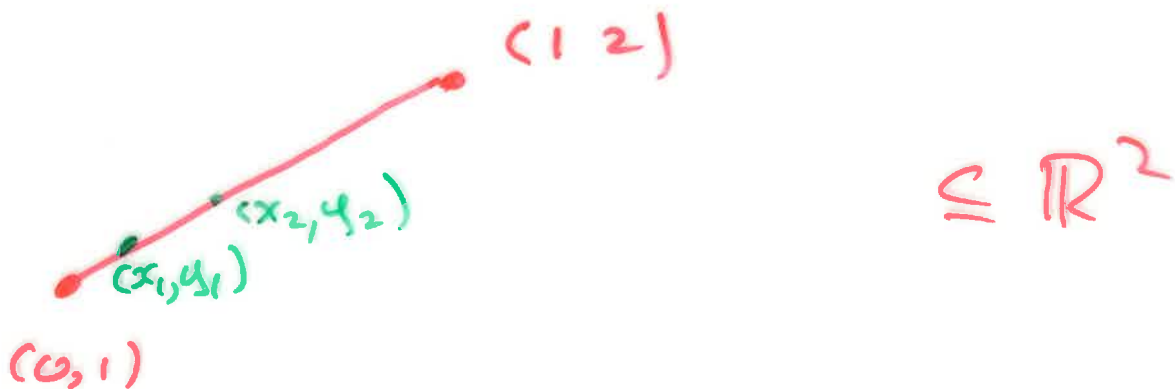
$$\|(x, y)\| = \sqrt{x^2 + y^2}.$$

Example Let S be the line segment from $(0, 1)$ to $(1, 2)$. Evaluate

$$L = \int_S (x^2 - y) dx + (y^2 + x) dy.$$

Soln

The line $y = x + 1$



passes through $(0, 1)$ and $(1, 2)$

$$P = \{ (x_0, y_0), (x_1, y_1), \dots, (x_n, y_n) \}$$

$$= \{ (x_0, x_0+1), (x_1, x_1+1), \dots, (x_n, x_n+1) \}$$

$$L = \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n A(x_i, y_i)(x_i - x_{i-1}) + B(x_i, y_i)(y_i - y_{i-1})$$

$$= \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n (x_i^2 - x_i - 1)(x_i - x_{i-1}) + ((x_i + 1)^2 + x_i)(x_i - x_{i-1})$$

$$= \int_0^1 (x^2 - x - 1 + (x+1)^2 + x) dx$$

$$= \int_0^1 (2x^2 + 2x) dx$$

= ...

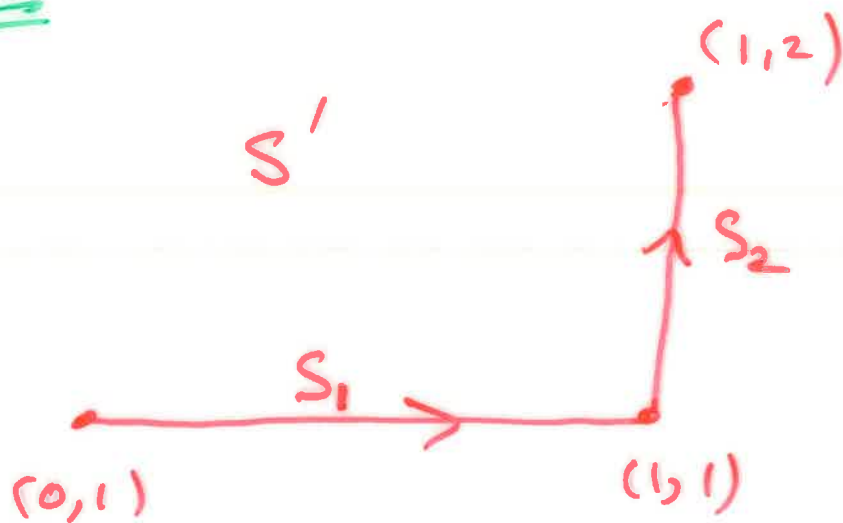
= $\frac{2}{3}$

Example Evaluate

$$L' = \int_{S'} (x^2 - y) dx + (y^2 + x) dy$$

where S' is the line from $(0, 1)$ to $(1, 1)$, followed by the line from $(1, 1)$ to $(1, 2)$.

Solⁿ



$$L' = \int_{S_1} (x^2 - y) dx + (y^2 + x) dy$$

$$+ \int_{S_2} (x^2 - y) dx + (y^2 + x) dy$$

The line $y = 1$

contains S_1

" " $x = 1$

" " S_2

$$L' =$$

$$\lim_{\|P\| \rightarrow 0} \sum_{i=1}^n (x_i^2 - 1)(x_i - x_{i-1}) + 0$$

$$+ \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n 0 + (y_i^2 + 1)(y_i - y_{i-1})$$

$$= \int_0^1 (x^2 - 1) dx + \int_1^2 (y^2 + 1) dy$$

$$= \dots$$

$$= -2.$$

Example Work is represented
by the 1-form

$$\omega = (3x - 4y + 2z) dx \\ + (4x + 2y - 3z^2) dy \\ + (2xz - 4y^2 + z^3) dz$$

Find the work done in
moving a particle once around
the following ellipse in the
 xy -plane, in the anti-clockwise
direction.

