First in-class test: Wed 4 October

Example A particle is moving in a constant force field. It takes 3 units of work to move the particle from point (x, y) to point $(x+1, y)$. It takes 4 units of work to move the particle from point (x, y) to point $(x, y+1)$.

We say that work is represented by the 1-form

$$w = 3\,dx + 4\,dy.$$

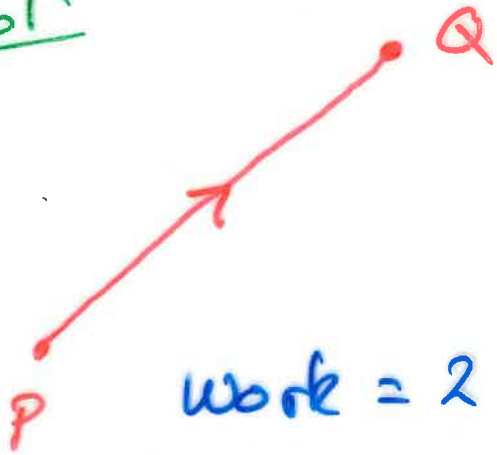
This is an example of a constant force field.

Example Consider a particle in a constant force field, with ~~work~~ given by the 1-form

$$\omega = 2dx + 3dy + 5dz.$$

Calculate the work done in moving the particle along the straight-line segment from point $P = (-1, 3, -5)$ to point $Q = (3, -1, 7)$.

Soln



$$Q - P = (4, -4, 12)$$

$$\text{work} = 2(4) + 3(-4) + 5(12)$$

$$= 56.$$

Example An investment portfolio involves two types of assets: type X and type Y.

It costs €3 to acquire one unit of asset X, and €-3 to relinquish one unit of asset X. It costs €4 to acquire one unit of asset Y and €-4 to relinquish one unit of Y.

We say that the marginal costs are represented by the 1-form

$$\omega = 3\,dx + 4\,dy .$$

Example Find the 1-form

$$\omega = A\,dx + B\,dy + C\,dz$$

describing work in the constant force field, where displacement of a particle from

$(0,0,0)$ to $(4,0,0)$ needs 3 units of work

$(1,-1,0)$ to $(1,1,0)$ " 2 "

$(0,0,0)$ to $(3,0,2)$ " 5 " .

Soln

$$3 = A \cdot 4$$
$$2 = B \cdot 2$$
$$5 = A \cdot 3 + C \cdot 2$$

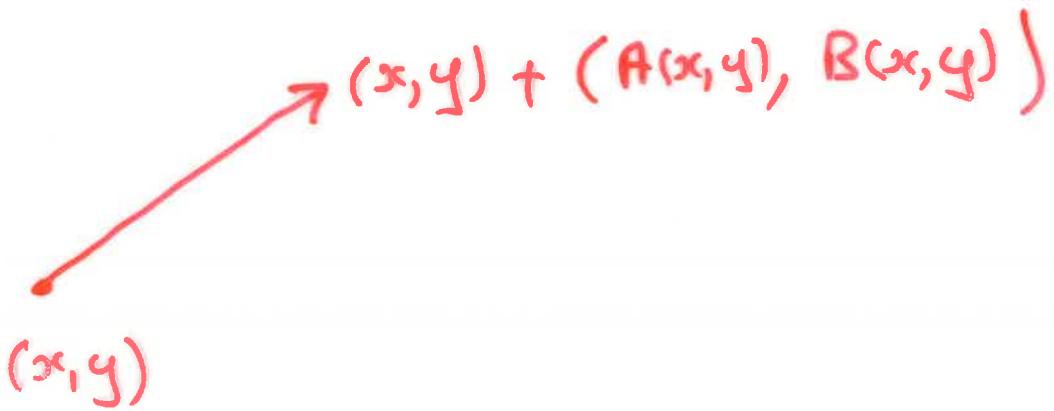
$$A = \frac{3}{4}$$
$$B = 1$$
$$C = \frac{11}{8}$$

We can think of a 1-form

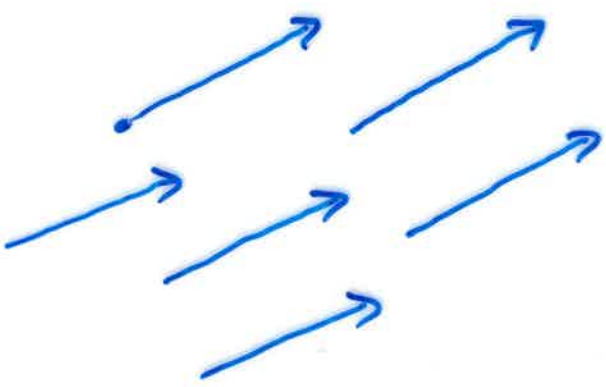
$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

as a collection of arrows in space (= plane for two variables).

For each point (x,y) in the space we have an arrow



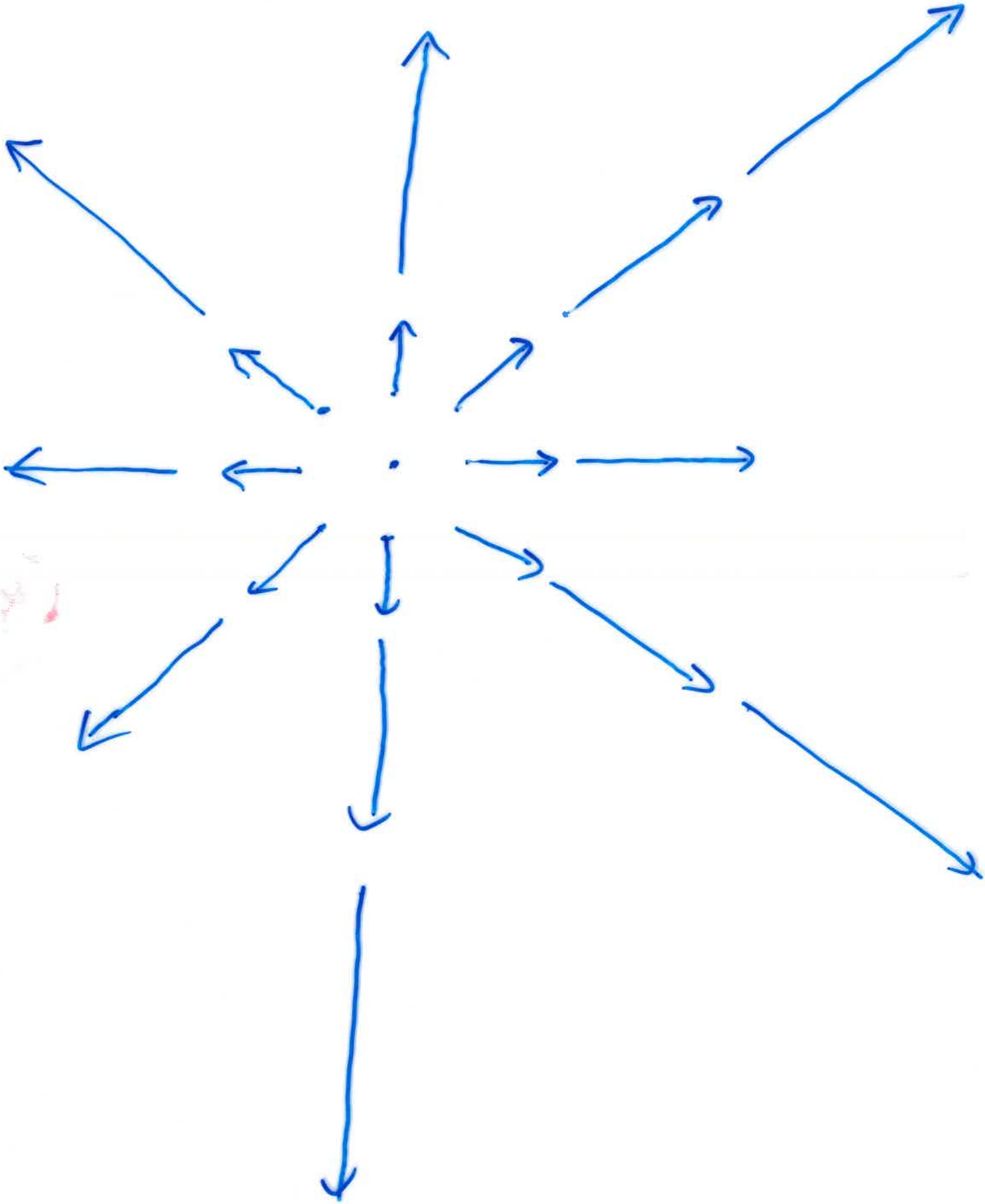
Example The 1-form $\omega = 2dx + dy$ can be pictured as



Example The 1-form

$$\omega = x\,dx + y\,dy$$

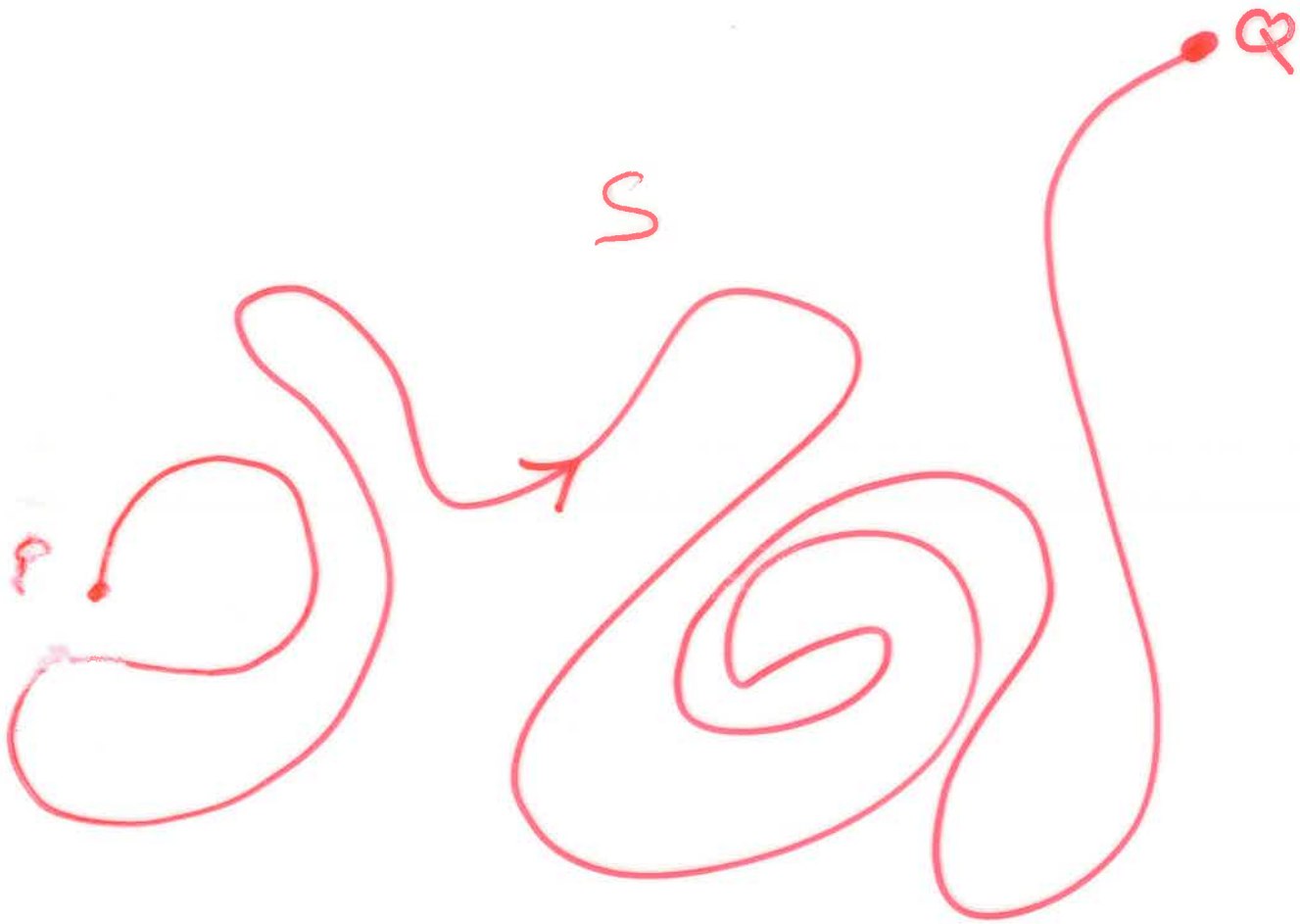
can be pictured as



Integration of 1-forms

Let $\omega = A(x,y)\,dx + B(x,y)\,dy$ be a differential 1-form.

Let $S \subseteq \mathbb{R}^2$ be a 1-dimensional, oriented, connected subset



Informally: If we think of ω as a "work 1-form" then

$$\int_S A(x,y)\,dx + B(x,y)\,dy$$

is the total work done in moving a particle from P to Q along S.