Tutorials:

Tuesday 6pm ADB-1020

Friday 12pm CA11B (Cairns Bldg)

Continued from last lecture

$$\cos(x) = \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$= \frac{1}{1+u^2} - \frac{u^2}{1+u^2}$$

$$= \frac{1-u^2}{1+u^2}$$

So

$$w = \int \frac{1}{5+3\cos(x)}\,dx$$

$$= \int \frac{1}{5+3\left(\frac{1-u^2}{1+u^2}\right)} \frac{2}{(1+u^2)}\,du$$

$$= \dots$$

$$= \int \frac{1}{4+u^2}\,du$$

from the log book

$$\omega = \frac{1}{2}\tan^{-1}\frac{u}{2} + C$$

$$\boxed{\omega = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\tan\left(\frac{x}{2}\right)\right) + C\,.}$$

Differential 0-forms on n-dimensional space

A differential 0-form on 2-dimensional space is a real valued function

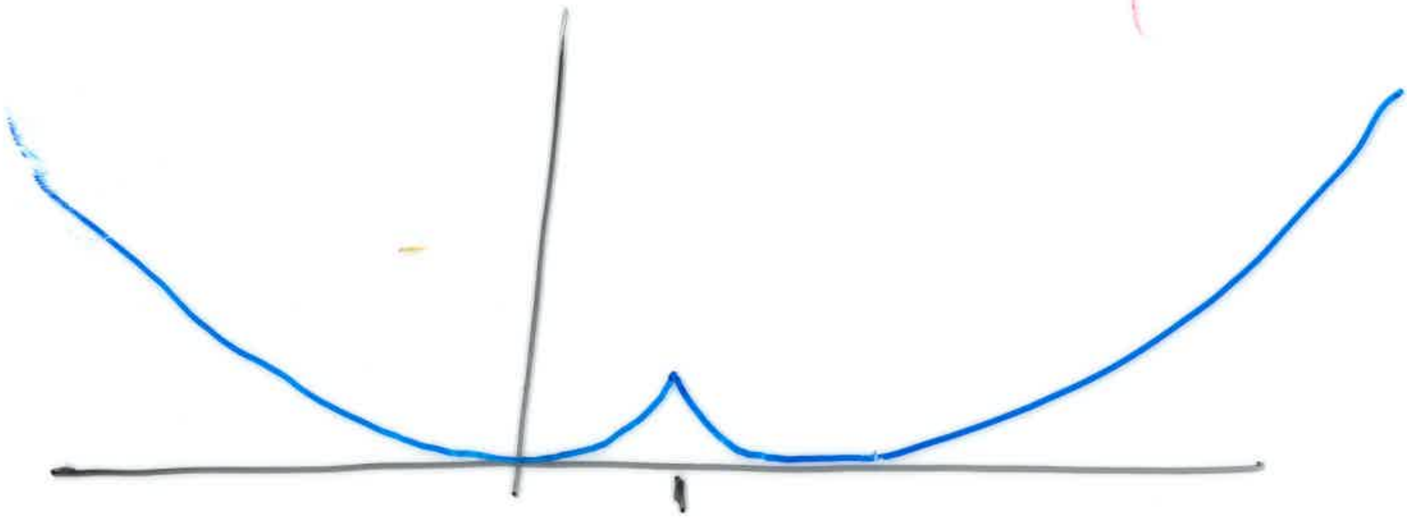
$$\omega = f(x, y)$$

which is "differentiable".
To explain this term, recall:

Informally: A function $f(x)$ is differentiable at a point x if the curve $y = f(x)$ has a well-defined (= unique) tangent line at x.

Example

$$y = \begin{cases} x^2 & x \le 1 \\ (x-1)^2 & x > 1 \end{cases}$$



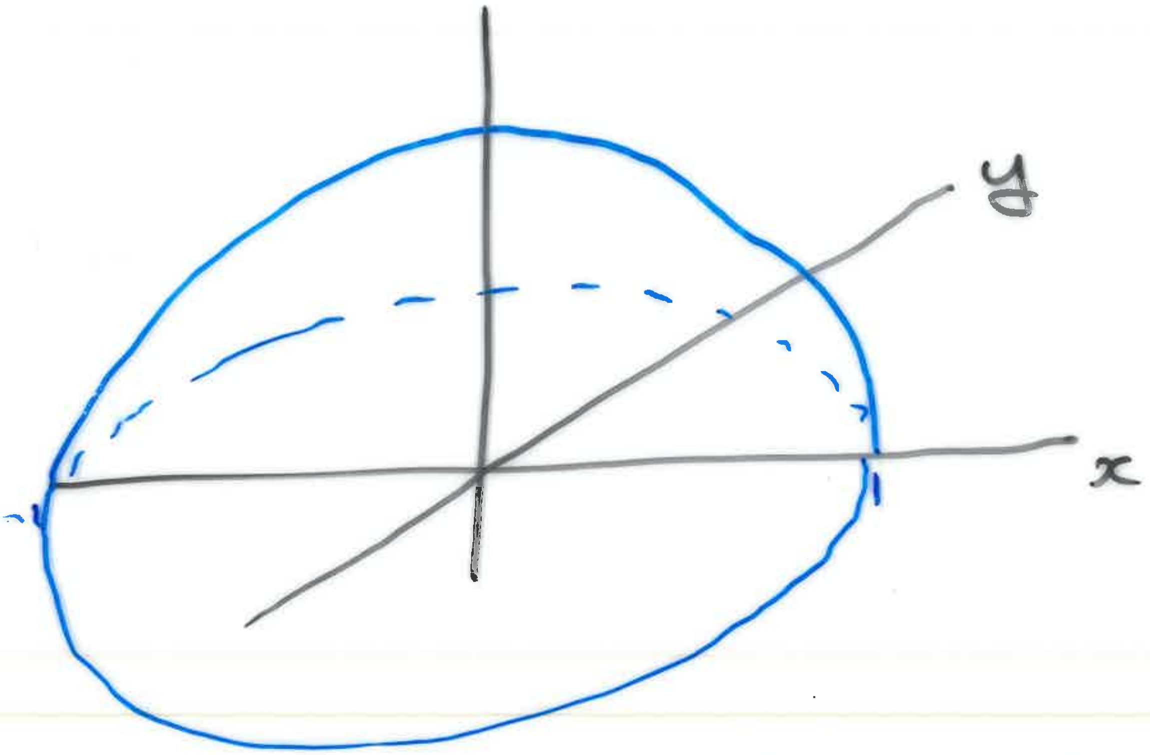
This is not differentiable at $x = 1$.

Informally A function $f(x,y)$ is differentiable at a point (x,y) if the surface

$$z = f(x,y)$$

has a well-defined (= unique) tangent plane at (x,y).

Example $z = \sqrt{1-x^2-y^2}$ is defined for $x^2+y^2 \leq 1$, and describes a surface.



For any point (x,y) in

$$S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$$

the surface has a well-defined tangent plane.

So

$$\omega = \sqrt{1-x^2-y^2}$$

is a differential 0-form

on S.

Let's skip the formal definition of differentiability

Differential 1-forms on n-dimensional space.

A differential 1-form on a 2-dimensional region S is a function

$$\omega = A(x,y)\,h_1 + B(x,y)\,h_2$$

that inputs a vector (x,y) and a vector (h_1,h_2) and

Here $A(x,y)$ and $B(x,y)$ are differentiable real valued functions.

Example Evaluate the 1-form

$$\omega = (x^2+y^2)h_1 + 2xy\, h_2$$

at $(x,y) = (2,4)$ and

$$(h_1, h_2) = \left(\tfrac{1}{4}, \tfrac{1}{4}\right).$$

Sol^n 9

Notation We usually denote

$$\omega = A(x,y)h_1 + A(x,y)h_2$$

by

$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

Example Evaluate the 1-form

$$\omega = (x^2+y^2)\,dx + 2xy\,dy$$

at $(x,y) = (2,4)$, $(dx,dy) = (\frac{1}{4}, \frac{1}{4})$.

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