Tutorials start this week.

Tue 6, Fri 12

$$\int_{\partial S} \omega = \int_S d\omega \qquad (*)$$

for $p=0$ and $n=1$, i.e. 0-forms ω in 1 variable, we understand all terms in $(*)$ except for $d\omega$.

Defn For a differential 0-form $\omega = F(x)$ we define the 1-form

$$d\omega = F'(x)\,dx$$

we call $d\omega$ the total derivative of ω, or just the derivative of ω.

So for $p=0$, $n=1$ we see that $(*)$ is just the Fundamental Theorem of Calculus.

Let's recall from 1st year

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(\nu_i)(x_i - x_{i-1})$$

where

- $P = \{a = x_0, x_1, \ldots, b = x_n\}$
- $\|P\| = \max\limits_{1\le i\le n} |x_i - x_{i-1}|$
- $\nu_i \in [x_{i-1}, x_i]$

<u>Proof of the Fundamental Theorem of Calculus</u>

Suppose that the Galway to Dublin train has a function speedometer, but a broke mileometer. The driver has a clock.

To estimate the distance travelled from $t=a$ to time $t=b$ the driver could calculate

$$\sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

where $f(t)$ is the speed of the train at time t, and

$a = t_0 < t_1 < \dots < t_n = b$

Let

$F(t) =$ total distance travelled at time t.

now

$$f(t) = F'(t)$$

and roughly

$$F(b) - F(a) \simeq \sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

Taking limits as $\|P\| \to 0$

$$F(b) - F(a) = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(t_i)(t_i - t_{i-1})$$

Thus

$$\boxed{F(b) - F(a) = \int_a^b f(t)\, dt}$$

or

$$\boxed{\int_{\partial S} \omega = \int_S d\omega}$$

with $\omega = F(t)$

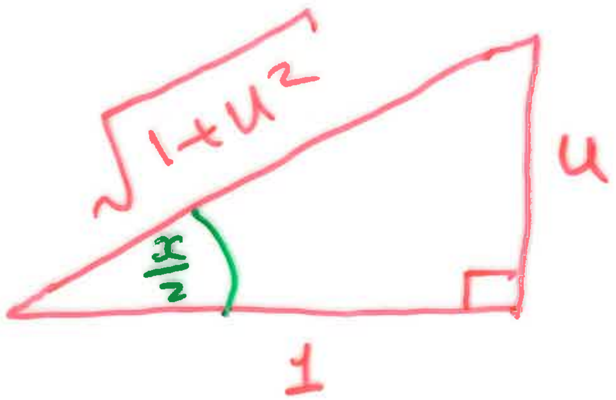
Example Find a differential 0-form ω whose total derivative is

$$d\omega = \frac{1}{5+3\cos(x)}\,dx.$$

Sol^n Using the language of 1st year maths, we want to find

$$\omega = \int \frac{1}{5+3\cos(x)}\,dx.$$

Let $u = \tan\left(\frac{x}{2}\right)$



$$\sin\left(\frac{x}{2}\right) = \frac{u}{\sqrt{1+u^2}}$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+u^2}}$$

$$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) dx$$

Now $dx = 2\cos^2\left(\frac{x}{2}\right) du$

$$= 2\frac{1}{1+u^2} du$$

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