A 1-form

$$\omega = A\,dx + B\,dy + C\,dz$$

is something we can integrate over a 1-dimensional oriented region. 1-forms represent abstract notions related to force fields or marginal cost. But their integral $\int_S \omega$ is something easy to understand: total work, total cost, ... .

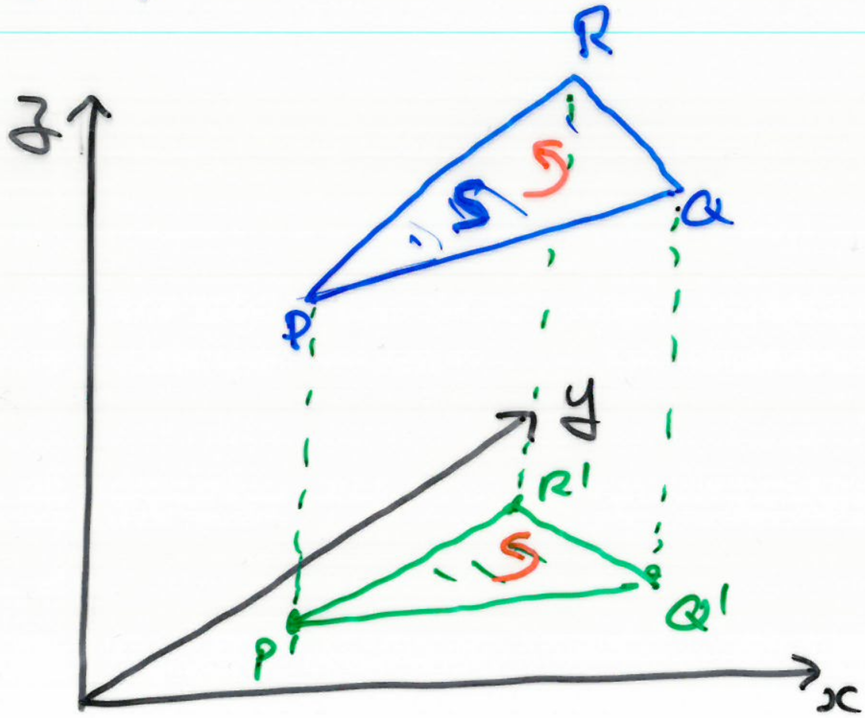
A 2-form

$$\omega = A\,dx \wedge dy + B\,dy \wedge dz + C\,dz \wedge dx$$

is something that can be integrated over a 2-dimensional oriented region.

# Constant 2-forms

Let $S$ be an oriented triangle in $\mathbb{R}^3$.




Let $S_z$ be the image of $S$ in the $xy$-plane under the projection

$$P_z : \mathbb{R}^3 \longrightarrow \mathbb{R}^2, \ (x,y,z) \longmapsto (x,y)$$

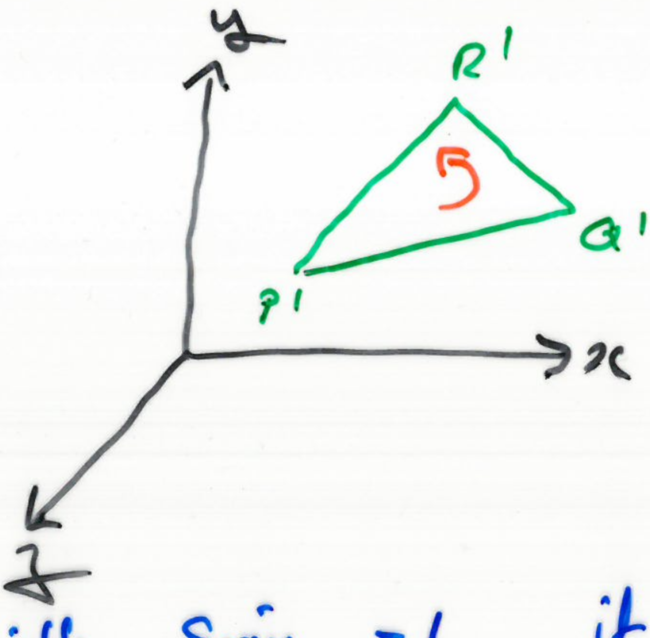
For any constant $A \in \mathbb{R}$ let

$$\int_S A \, dx \wedge dy$$

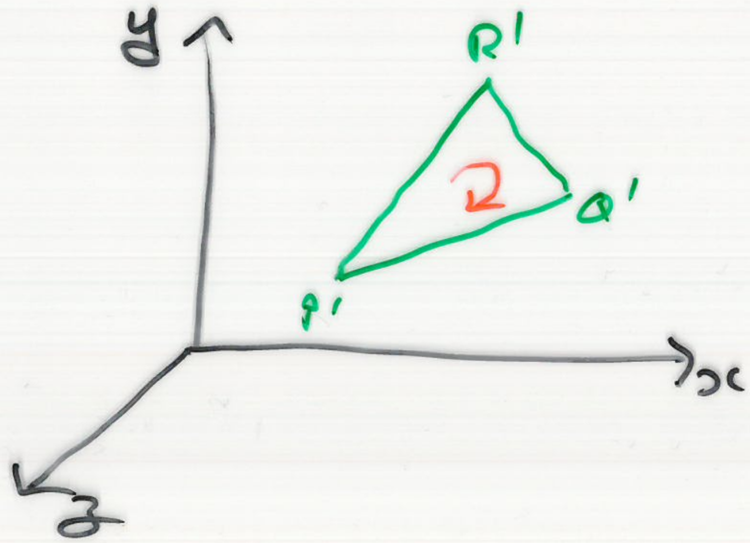
denote

$\pm A \times (\text{area of } S_z)$

with sign $+1$ if




and with sign $-1$ if



Similarly define for constants $B, C \in \mathbb{R}$

$$\int_S C \, dz \wedge dx$$

and

$$\int_S B \, dy \wedge dz \,.$$

Always: $dx \wedge dy$, $dy \wedge dz$, $dz \wedge dx$


**Defn**

$$\int_S A \, dx \wedge dy + B \, dy \wedge dz + C \, dz \wedge dx$$

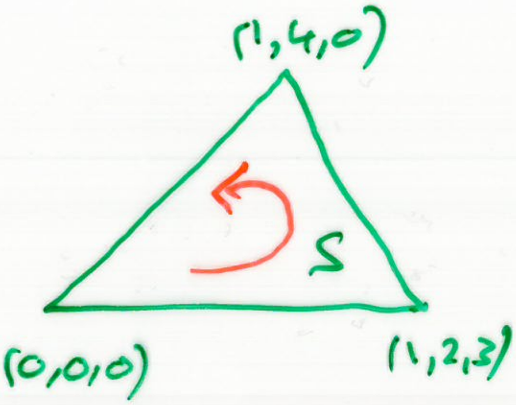
$$= \int_S A \, dx \wedge dy + \int_S B \, dy \wedge dz + \int_S C \, dz \wedge dx$$

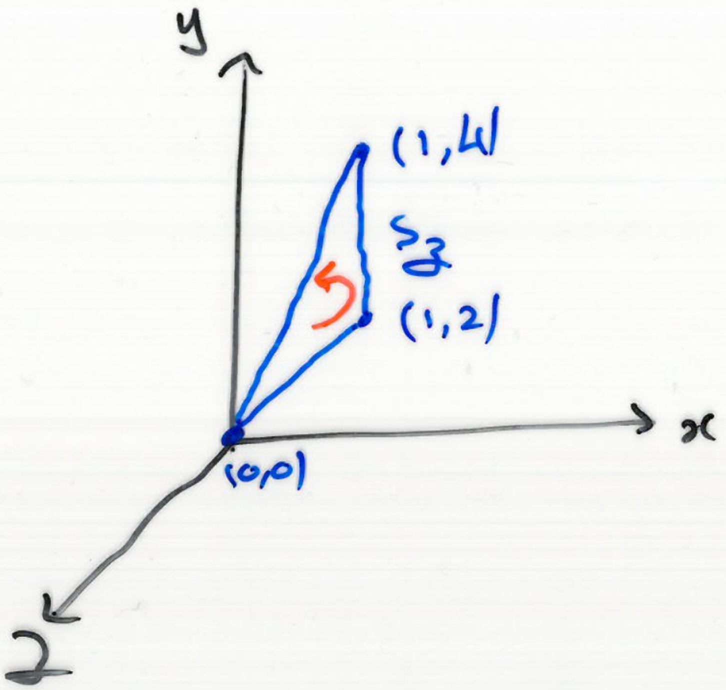
__Example__ Evaluate

$$I = \int_S dx \wedge dy + 3\, dz \wedge dx$$

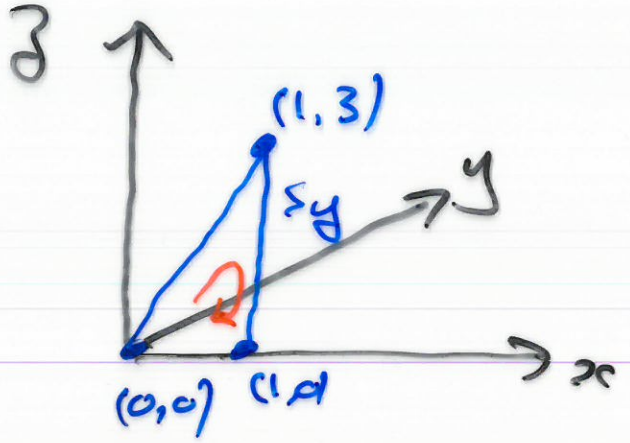
over the oriented planar triangle $S$ with vertices $(0,0,0)$, $(1,2,3)$ and $(1,4,0)$ in that order.

__Sol$^n$__ $I = \int_S dx \wedge dy + \int_S 3\, dz \wedge dx$




$$\text{area of } S_z = \frac{1}{2}\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 1$$




area of $S_y = \frac{3}{2}$

$$I = +1.1 + +3.\frac{3}{2}$$
$$= \frac{11}{2}.$$

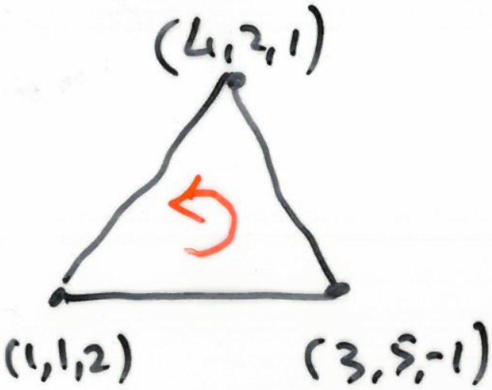
## Example

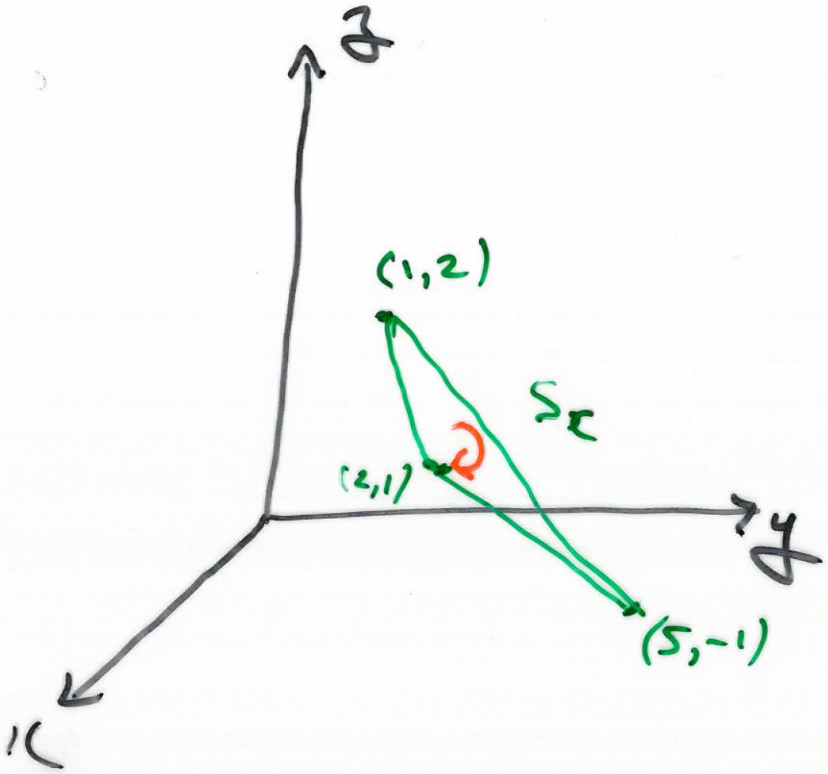
Evaluate

$$I = \int_S dy \wedge dz$$

where $S$ is the oriented planar triangle with vertices $(1,1,2)$, $(3,5,-1)$ and $(4,2,1)$ in that order.

Sol$^n$




area of $S_x$ $= -\frac{1}{2}\begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} = \frac{1}{2}$

$$I = \int_S dy \wedge dz = -\frac{1}{2}$$