# MA2286 Differential Forms

(= Calculus)

Aim: Explain and apply the generalized Stokes formula:

$$\int_{\partial S} \omega = \int_S d\omega$$

where

- $\omega$ is a differential p-form in $n$ variables
- $S$ is a nice region in $\mathbb{R}^n$
- $\partial S$ is the boundary of $S$.
- $\int$ is an integral

# Differential 0-forms in 1-variable

$(p=0,\ n=1)$

A differential 0-form in 1 variable is just a differentiable real valued function

$$\omega = f(x)$$

Examples

$$\omega = 3x - 4$$

$$\omega = 3x^2 + 4$$

$$\omega = \sin(x)$$

are differential 0-forms.

usually a differential 0-form is given in the context of some closed interval

$$S = [a,b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1,b_1] \cup [a_2,b_2] \cup \dots \cup [a_k,b_k].$$

We only require $\omega$ to be differentiable on $S$.

## Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 1000].$$

Clearly $\omega$ is <u>not</u> a differential 0-form on

$$S = [-1, 1]$$

## Terminology

I'll say <u>0-form</u> instead of differential 0-form.

For $a < b \in \mathbb{R}$ we write

$$[a, b] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

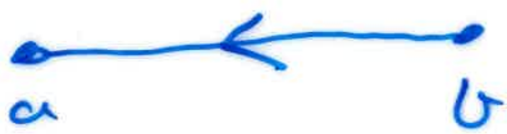
and we picture this as

The arrow is an **orientation** that specifies the direction of travel for $a$ to $b$.

for $a < b \in \mathbb{R}$ we write

$$[b, a] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and picture this as




We say that $[a, b]$ and $[b, a]$ are **oriented intervals**.

**Example** $S = [2,1] \cup [3,4] \cup [6,5]$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The **boundary** of the oriented interval $S = [a, b]$ is the set

$$\partial S = \{a, b\}$$

the set consisting of two points, the initial point $a$ and final point $b$.

Example

$$S = [2,1] \cup [3,4] \cup [6,5]$$

$$\partial S = \{1, 2, 3, 4, 5, 6\}.$$

Terminology we'll say that $S = [a,b]$ is 1-dimensional, and that the boundary is 0-dimensional.

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = F(b) - F(a).$$

Example Integrate the differential 0-form

$$\omega = 3x^2 + 4$$

over the boundary of the oriented interval

$$S = [2, 1].$$

Sol$^n$

$$\begin{aligned}\int_{\partial S} \omega &= \omega(1) - \omega(2) \\ &= 3(1^2)+4 - (3(2^2)+4) \\ &= -9.\end{aligned}$$