SAMPLE

Exam Code(s)	3BA1, 4BA4, 4BHR1, 1EM1, 4BTP1, 3BS9.
Exam(s)	Third Arts and Science.
Module(s)	Complex Variables.
Module Code(s)	MA302.
Paper No	1.
Repeat Paper	No
External Examiner(s)	Prof. External
Internal Examiner(s)	Prof. G. Ellis, Dr. M. Hayes.
Instructions:	Full marks for **three** questions. **All questions carry equal marks.**
Duration	2 hours.
No. of Pages	4 pages.
Department(s)	Mathematics.
Course Co-ordinators(s)	
Requirements:	
Release in Exam Venue	Yes ☑ No ☐
MCQ	Yes ☐ No ☑
Handout	
Statistical Tables/ Log Tables	Yes.
Cambridge Tables	
Graph paper	
Log Graph Paper	
Other Materials	Students may use their own electronic calculators which must not be capable of storing text.

p.t.o.

1. (a) Express

$$\frac{2+3i}{6+8i} - \frac{-i}{6-8i},$$

in the form $x + iy$ with real numbers x and y.

(b) Differentiate

$$f(z) = (4z^3 - 5iz^2)\sin^4(z).$$

Without using the Cauchy Riemann equations state the region in the Argand diagram where the function $f(z)$ is differentiable.

(c) Use the Cauchy-Riemann equations to find all points $z = x + iy$, where

$$f(z) = (x^5 - 10x^3y^2 + 5xy^4) + i(5x^4y - 10x^2y^3 + y^5),$$

is differentiable and find $f'(z)$ for those z.

(d) Find where

$$g(z) = \mathrm{Im}(z^2),$$

(the imaginary part of z^2) is differentiable in the complex plane.

(e) What does it mean for a function $u(x, y)$ to be harmonic? Show that

$$u(x, y) = -x + 9y - e^x \cos(y),$$

is harmonic and find a harmonic conjugate v of u.

p.t.o.

2. (a) Solve the equation

$$z^6 + 1 = i\sqrt{3},$$

and sketch all the solutions in an Argand diagram.

(b) Evaluate the complex integral:

$$\int_C [z\mathrm{Re}(z) - \bar{z}\mathrm{Im}(z)]dz,$$

where C is the line segment joining -1 to i.

(c) Evaluate the complex integral:

$$\int_C [iz^2 - z - 3i]dz,$$

where C is the quarter circle with centre the origin which joins -1 to i.

p.t.o.

3. (a) State Cauchy's integral formula for derivatives and use it to evaluate

$$\int_C \frac{5z^5 + 3z^3}{(z+1)^3} dz,$$

where C is the positively oriented circle $|z| = 2$.

(b) Expand

$$f(z) = \frac{3}{(z-1)(z-2)},$$

in a Laurent series valid in (i) $0 < |z-1| < 1$ and (ii) $|z-1| > 1$.

(c) State the Residue Theorem and use it to evaluate:

$$\int_C \frac{1}{(z-i)^2(z^2+4)},$$

where C is the positively oriented circle $|z-i| = 2$.

End of paper.