Lect 8
Ex ctd

Compute $\displaystyle\int_{\theta=0}^{\pi}\int_{r=1}^{2} 3r^2\cos\theta + 4r^3\sin^2\theta \, dr\, d\theta$

Inner $= \displaystyle\int_{r=1}^{2} 3r^2\cos\theta + 4r^3\sin^2\theta \, dr = \left[\frac{3r^3}{3}\cos\theta + 4\sin^2\theta\frac{r^4}{4}\right]_1^2$

$= (8\cos\theta + 16\sin^2\theta) - (\cos\theta + \sin^2\theta)$

$= 7\cos\theta + 15\sin^2\theta$ (= function of θ)

Outer: $\displaystyle\int_{\theta=0}^{\pi} 7\cos\theta + 15\sin^2\theta \, d\theta$

⚠ tricky one?

⚠ Trig Identity for $\sin^2$

$= \displaystyle\int_{\theta=0}^{\pi} 7\cos\theta + 15\left(\frac{1}{2}(1-\cos 2\theta)\right) d\theta$

$= \left[7\sin\theta + \frac{15}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]_0^{\pi}$

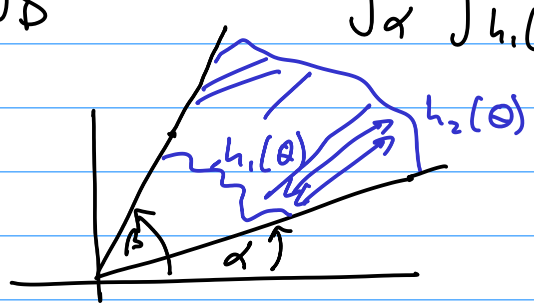
$= \left(\underbrace{7\sin\pi}_{0} + \frac{15}{2}\left(\pi - \frac{\sin 2\pi}{2}\right)\right) - \underbrace{\left(7\sin 0 + \frac{15}{2}\left(0 - \sin 0\right)\right)}_{0}$

$= \dfrac{15\pi}{2}$

Thm: If f is cts on a polar region of the form $D = \{(r,\theta) : \alpha \le \theta \le \beta,\ h_1(\theta) \le r \le h_2(\theta)\}$

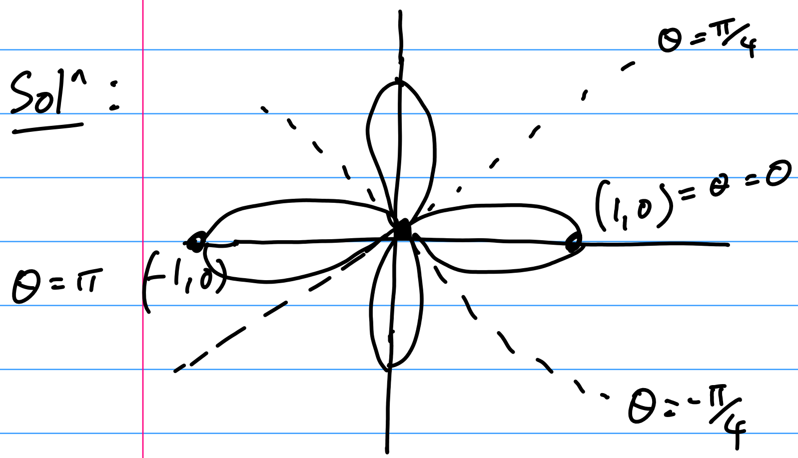
then $\displaystyle\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$

e.g.



Example: (Stewart) Use a double integral to find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$

Sol^n:



For one loop, we take θ ranging between $-\pi/4$ and $\pi/4$

⚠ and r ranges between 0 and $\cos 2\theta$

ie $-\pi/4 \le \theta \le \pi/4$
$0 \le r \le \cos 2\theta$

Area is thus $\iint_D 1 \, dA$ ⚠ uniform function

$$= \int_{-\pi/4}^{\pi/4} \int_0^{\cos 2\theta} 1 \cdot r \, dr \, d\theta$$ cf theorem above

= (exercise) $= \ldots = \dfrac{\pi}{8}$