

MA133C & MA160
Calculus 1

Lecture 22

Recap on differentiation by rule

▶ **Rule for the derivative of a power**

$$\text{If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}$$

▶ **Derivatives of exponential and trigonometric functions:**

$$\frac{d}{dx}(\sin(x)) = \cos(x); \quad \frac{d}{dx}(\cos(x)) = -\sin(x); \quad \frac{d}{dx}(\tan(x)) = \frac{1}{(\cos(x))^2}$$

$$\frac{d}{dx}e^x = e^x$$

▶ **Sum, difference, product with a constant c**

$$(f + g)' = f' + g'; \quad (f - g)' = f' - g'; \quad (c \cdot f)' = c \cdot f'$$

▶ **Product and quotient rules**

$$(fg)' = f'g + fg'; \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

▶ **Chain rule**

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

Problems

Find the derivative of each of the following functions. Also, find an equation for the tangent to the graph of $h(x)$ at its point with $x = 0$. Same for $u(x)$ at its point with $x = 1$.

1. $f(x) = \sqrt{2x} + 2 \cos(x)$.

2. $g(x) = 5 + xe^{x^2+1}$.

3. $h(x) = \frac{2}{x^2 + 3}$.

4. $u(x) = \frac{x^2 - 1}{2x - 3}$.

5. $v(x) = \sqrt{x} - \sin(x)$.

Recap: L'Hopital's Rule

L'Hopital's Rule

1. Suppose f and g are differentiable around a point a , that $g'(x) \neq 0$ near a (except possibly at a) and that at a we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

2. Suppose f and g are differentiable when x is large (or large negative) and that at infinity we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then:

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}.$$

Problems

Evaluate the following limits.

1. $\lim_{x \rightarrow 3} \frac{x^3 - 5x^2 + 6x}{2x^2 - 6x}$.

2. $\lim_{\theta \rightarrow 0} \frac{\sin(5\theta) + \theta}{\sin(4\theta)}$.

3. $\lim_{x \rightarrow +\infty} (1+x)e^{-x^2}$.

4. $\lim_{x \rightarrow 1/2} \frac{6x^2 + 5x - 4}{4x^2 + 16x - 9}$.