# MA133C & MA160

# Calculus 1

# Course info

- **Course:** Calculus 1 – MA133C & MA160
- **Lecturer:** Dr Angela Carnevale
- **Lectures:** Monday at 1pm in IT125 and Tuesday at 10am in Tyndall.
- **Office hours:** Mondays 3–4pm, Wednesdays 2–3pm, AdB–2003
- **Reading material:**
  - "Calculus: Early Transcendentals" by Stewart (available @ MAIN LIB 515 STE)
  - Notes from class (online)
- **Continuous Assesment:** There will be 6 online assignments.

  Links will be available on the webpage of the course and on blackboard. The continuous assesment for MA133 comprises 50% of the mark for MA131.
- **Final exam:** There will be a 2 hour exam, covering both algebra and calculus.

# Syllabus

- Functions and their graphs (approx. 8 lectures)

  Various ways to represent a function. Mathematical models & a catalog of essential functions: polynomial functions, exponentials, logarithms, inverse functions and their graphs.

- Derivatives (approx. 8 lectures)

  Tangents and velocity. Limits, continuity, asymptotes. Derivatives, differentiation rules.

- Local and global optima (approx. 8 lectures)

  Derivatives in use: applications to find maximum and minimum values. Optimisation problems.

# What are functions and why study them?

Informally, a **function** is a law arising whenever a quantity **depends** on another. Our goals are:

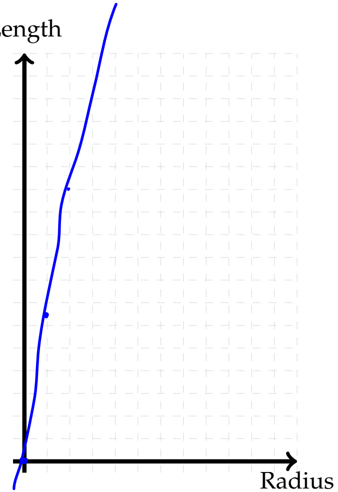
- Learn how to represent a functions
- Familiarise with a list of fundamental functions which occur in your studies & in real life applications
- Perform operations with and on functions
- Find their maxima, minima and local optima.

## Examples: encoding data into a function

What function of the radius is the length of a circle?

### Experimental evidence

| Radius | Length |
|---|---|
| 10 | ~62,8 |
| 20 | ~125 |
| 0 | 0 |
| … | … |




By the way, this is also one way to compute a rough approximation of the number $\pi$.

## Examples: description of a function with words

Functions can be described in words:

"Assign to any number one less than half of its square."

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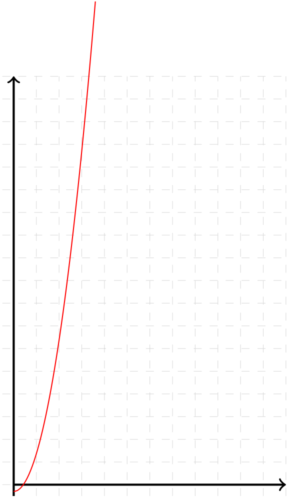
How would you write this as a function of a number $n$?

## Examples: graphical and algebraic description

Finally, functions can be given through their **graphs** or through **formulae**.



$$f(x) = \frac{1}{2}x^2 - 1$$

# Examples: graphical and algebraic description

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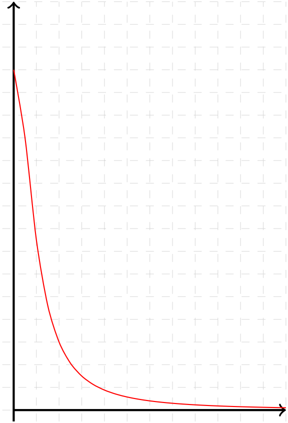


$$f(x) = \frac{15}{x^2 + 1}$$

# Is this a function?

- The area of a rectangle whose perimeter is 20, as a function of $x$, the length of one of its sides. ✓
- The length of the side of a square of given area $A$. ✓
- A real number whose square is $x$. ✗
- The temperature of a room at a given time.✓
- The time of the day at which a room has a certain temperature, as a function of the time. ✗
- Which person in this room was born on a certain weekday, as a function of the day. ✗

**Definition**

A **function** $f$ from a set $X$ to a set $Y$ is a law which associates to each $x$ in $X$ exactly one $y$ in $Y$. We write $f: X \longrightarrow Y$ and $f(x) = y$.

We will mostly consider functions of <u>one real variable</u> which are <u>real-valued</u>. This means:

$f : \mathbb{R} \to \mathbb{R}$ (remember: $\mathbb{R}$ is the set of real numbers with the usual operations)

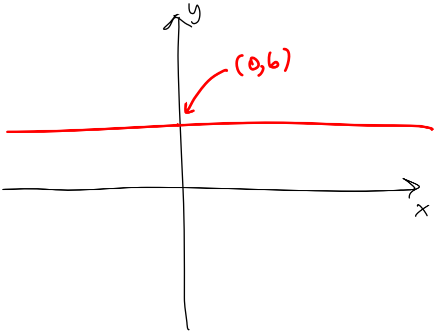
**<u>Example 0.1</u>** (Constant function)

For fixed $b \in \mathbb{R}$ consider

$f : \mathbb{R} \to \mathbb{R}$ defined by

$f(x) = b$

The graph of such function is



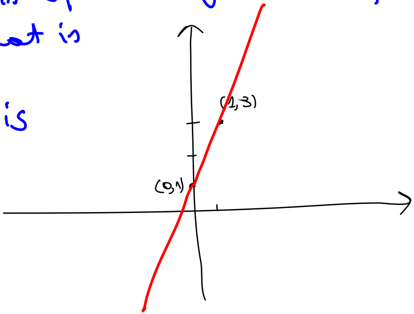
**Example 0.2** ## Linear functions

For fixed real numbers $a$ and $b$ we can define a linear function $f: \mathbb{R} \to \mathbb{R}$ as

$$f(x) = ax + b$$

Note that for $a = 0$ this is the constant function from before.
The graph of a linear function is a (non-vertical) line.
All we need to know to draw it is 2 points through which it passes.
For instance if $a = 2, b = 1$, that is

$f(x) = 2x + 1$ then the graph is




$f(0) = 1$

$f(1) = 2 \cdot 1 + 1 = 3$