

Defn: A Linear Transformation
(of the Plane) L is a function

$$L: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$v \longrightarrow L(v)$$

such that

$$1. L(u+v) = L(u) + L(v)$$

$$\forall u, v \in \mathbb{R}^2$$

$$2. L(kw) = kL(w)$$

$$\forall k \in \mathbb{R} \text{ \& } \\ \forall w \in \mathbb{R}^2.$$

How Do They Look:

$$v = (x, y) = \underbrace{x(1, 0)}_{(x, 0)} + y(0, 1)$$

Let L Be a linear Transformation

$$L(v) = ? \quad v = (x, y)$$

$$L((x, y)) = L(x(1, 0) + y(0, 1))$$

$$\stackrel{(1)}{=} L(x(1, 0)) + L(y(0, 1))$$

$$\stackrel{(2)}{=} x \underbrace{L(1, 0)} + y \underbrace{L(0, 1)}$$

$$= x(a, c) + y(b, d)$$

$$= (ax, cx) + (by, dy)$$

$$= (ax + by, cx + dy)$$

where $a, b, c, d \in \mathbb{R}^2$

Conversely and Rule $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

of the form

$$L(x, y) = (ax + by, cx + dy)$$

is a Linear Transformation.)

So $L \longleftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ a 2×2 Matrix

$$\underline{\underline{\text{Ex:}}}$$
 $L(x, y) = (3x + 4y, 7x - 6y)$

Is Linear ($a = 3, b = 4$
 $c = 7, d = -6$)

$$\underline{\underline{\text{Ex:}}}$$
 $L(x, y) = (2x + y, 7x + 6)$
 $(L(0, 0) = (0, 6) \neq (0, 0))$

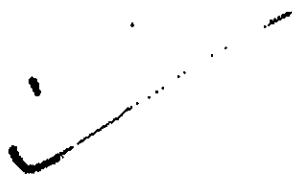
NOT Linear.

$$\underline{\underline{\text{Ex:}}}$$
 $L(x, y) = (xy + 2x, 3y + 6x)$

NOT Linear

$$\underline{\underline{\text{Ex:}}}$$
 $L(x, y) = (x + 2y, 3x + 4y)$

Linear.



Linear Transformations send
~~lines~~ to lines if L is
a line in \mathbb{R}^2 , then $L(l)$ is also
a line (or a point)

$$L(l) = L(P + tv) \quad (t \in \mathbb{R})$$

$$\stackrel{(1)}{=} L(P) + L(tv)$$

$$\stackrel{(2)}{=} \underbrace{L(P)}_{\text{a vector}} + t \underbrace{L(v)}_{\text{vector}} \quad t \in \mathbb{R}$$

↓
is the Parametric form of
the line through $L(P)$ in
the direction $L(v)$

(If $L(v) \neq (0, 0)$)