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<u>Defn:</u> A Linear Transformation (of the Plane) L is a function

$$L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$v \longrightarrow L(v)$$

such that

1. $L(u+v) = L(u) + L(v)$

$\forall\ u, v \in \mathbb{R}^2$

2. $L(kw) = k\,L(w)$

$\forall\ k \in \mathbb{R}$ &

$\forall\ w \in \mathbb{R}^2$.

How Do They Look:

$$v = (x,y) = x(1,0) + y(0,1)$$
$$= (x,0) + (0,y)$$

Let L Be a linear Transformation

$L(v) = ?$ $\qquad v = (x, y)$

$$L((x,y)) = L\big(x(1,0) + y(0,1)\big)$$

$$\overset{(1)}{=} L(x(1,0)) + L(y(0,1))$$

$$\overset{(2)}{=} x\underbrace{L((1,0))} + y\underbrace{L((0,1))}$$

$$= x(a,c) + y(b,d)$$

$$= (ax, cx) + (by, dy)$$

$$= (ax + by,\ cx + dy)$$

where $a, b, c, d \in \mathbb{R}^2$

Conversely and Rule $L : \mathbb{R}^2 \to \mathbb{R}^2$ of the form

$$\boxed{L((x,y)) = (ax + by,\ cx + dy)}$$

is a Linear Transformation.)

So $L \longleftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ a 2×2 Matrix

Ex: $L((x,y)) = (3x+4y,\ 7x-6y)$

Is Linear $(a = 3,\ 4 = 4$

$c = 7,\ d = -6)$

Ex: $L((x,y)) = (2x+y,\ 7x+6)$

$(L((0,0)) = (0,6) \neq (0,0)$

NOT Linear.

Ex: $L((x,y)) = (xy+2x,\ 3y+6x$

NOT LINEAR

Ex: $L((x,y)) = (x+2y,\ 3x+4y)$

Linear.

Linear Transformations send ~~lines~~ to lines i.e. if ℓ is a line in $\mathbb{R}^2$, then $L(\ell)$ is also a line (or a point)

$$L(\ell) = L(P + tv) \quad (t \in \mathbb{R})$$

$$\overset{(1)}{=} L(P) + L(tv)$$

$$\overset{(2)}{=} \underbrace{L(P)}_{\text{a vector}} + t\,\underbrace{L(v)}_{\text{vector}}, \quad t \in \mathbb{R}$$

is the Parametric form of the line through $L(P)$ in the direction $L(v)$

(If $L(v) \neq (0,0)$)