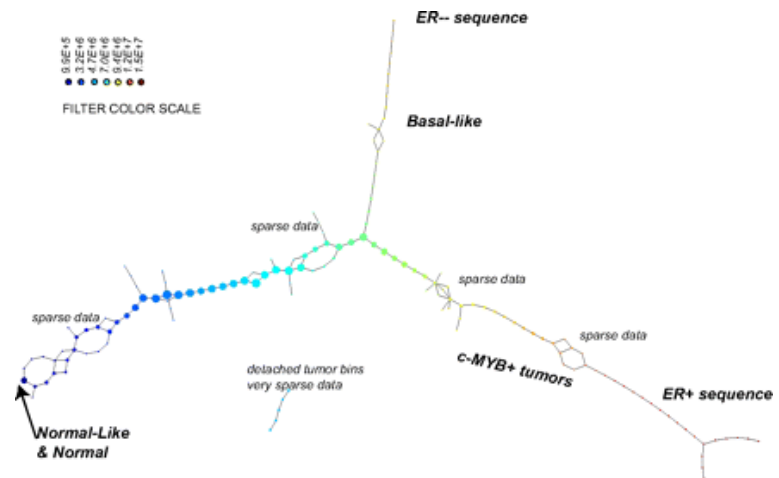


Breast cancer microarray gene expression data



Nicolau, Levine, Carlsson (PNAS, 2011): identified a subgroup of ER+ breast cancers. These patients exhibit 100% survival.

Part 2

PCA

$$\rho: \mathbb{R}^d \rightarrow \mathbb{R}^k$$

↓
Johnson-Lindenstrauss

Represents data points $v \in \mathbb{R}^d$ as points $\rho(v) \in \mathbb{R}^k$, with $k < d$.

Clustering $\rho: \mathbb{R}^d \rightarrow X$

Represents data points $v \in \mathbb{R}^d$ as points $\rho(v) \in X$ is a finite space of points.

In general, let's define dimension

reduction as any useful representation

$$\rho: \mathbb{R}^d \rightarrow X$$

with X a simplex (topological)

space.

Example Nicolae, Levine, Carlsson (2011)

295 samples from breast cancer tumors

15 samples from normal healthy breast tissue.

A dissimilarity matrix (310×310) was created and (somehow) used to create a graph X .

In the graph the nodes are "bins" containing samples. A

sample may lie in two

bins, in which case the bins are connected by an edge.

Bins are coloured:

Blue = similar to normal

Red = very different to normal

ER⁺ = Estrogen receptor positive

ER⁺ branch of graph had a good survival rate.

The c-Myb⁺ portion of the graph is defined as the region lying between two sparse regions

The graph was constructed using only the dissimilarity matrix.

However c-Myb⁺ region had 100% death rate.

Conclusion: c-Myb⁺ warrant being identified as a breast cancer group of genes.

Defn A simplicial complex consists of a set V and a collection K of certain subsets $\sigma \subseteq V$. The following conditions must hold:

- 1) $\{v\}$ is in the collection K for all $v \in V$.
- 2) If $\sigma \in K$ and if $\sigma' \subset \sigma$ then $\sigma' \in K$.

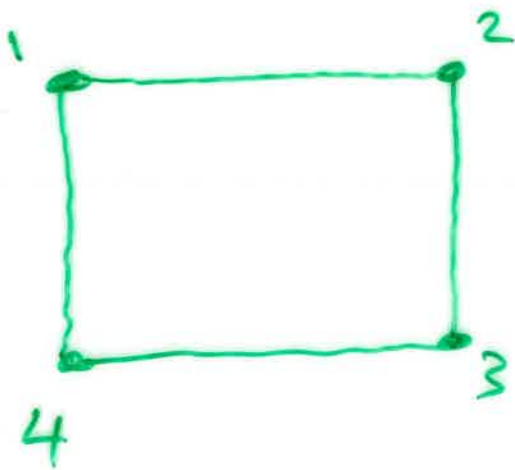
We call the elements $v \in V$ vertices, and the subsets $\sigma \in K$ simplices. We call $\sigma \in K$ an n -simplex if $\sigma = \{v_0, v_1, \dots, v_n\}$ consists of $n+1$ vertices.

Example $V = \{1, 2, 3, 4\}$

$K = \{ \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\} \}$

In this example we have only 0-simplices and 1-simplices.

We call 0-simplices vertices, and 1-simplices edges. We can picture K as a graph



Example 2 $V = \{1, 2, 3, 4, 5, 6\}$

$K = \{ \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\},$
 $\{1, 2\}, \{1, 3\}, \{2, 3\}, \{2, 4\}$
 $\{4, 5\}, \{4, 6\}, \{5, 6\},$
 $\{1, 2, 3\}, \{4, 5, 6\} \}$

we have six 0-simplices, seven
1-simplices, and two 2-simplices,
we picture K as

