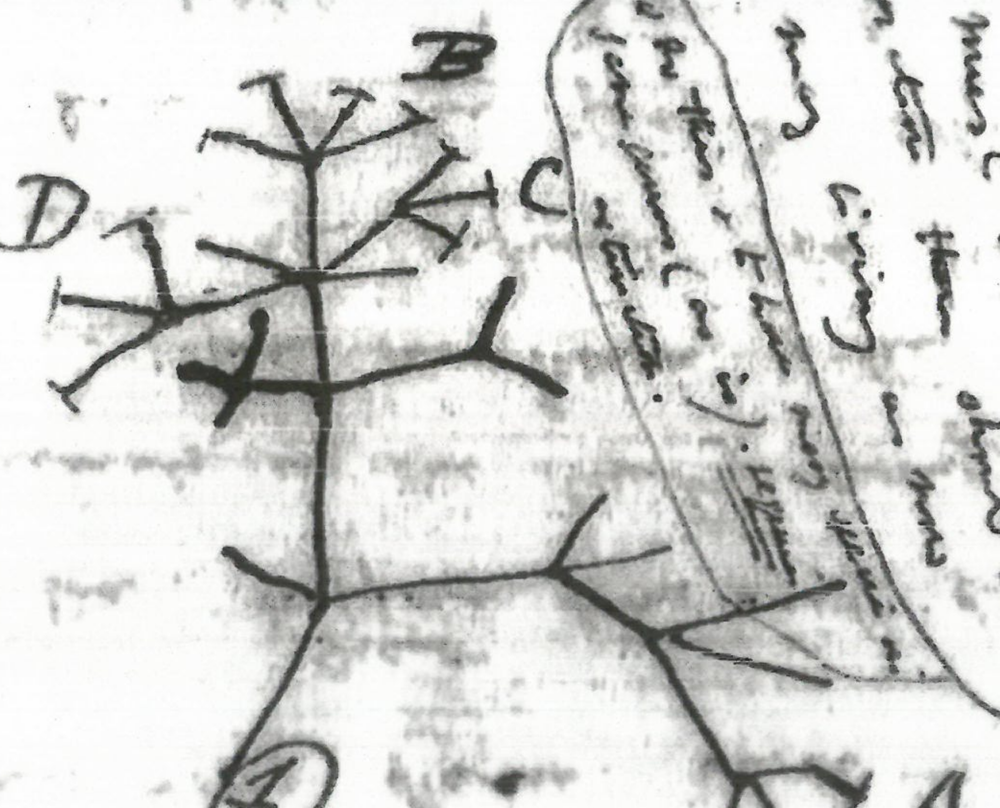
I think



Case must be that one generation then should be as many living as now. To do this & to have as many species in same genus (as is) requires extinction.

Introduction to topological data analysis

	H	M	R	C	W
H	0	11	10	14	22
M	11	0	3	13	21
R	10	3	0	12	20
C	14	13	12	0	16
W	22	21	20	16	0

- Human, Mouse, Rat, Cat, Whale
- Halford, M&S, River Island, Currys, Woodies

$\mathrm{dist}(H,H) = 0$

$\mathrm{dist}(H,M) = \mathrm{dist}(M,H)$

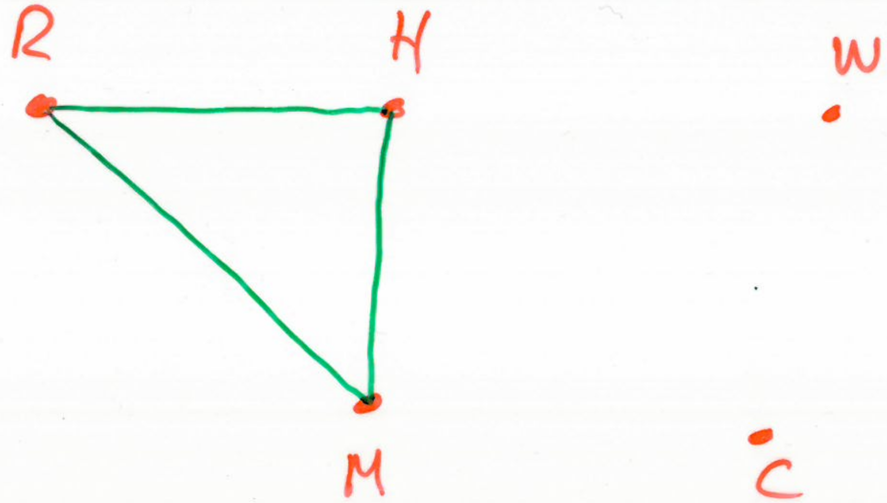
Choose $\varepsilon > 0$, called a threshold, and consider the graph G_ε with vertices

H, M, R, C, W

and with a edge

if $\mathrm{dist}(X,Y) \leq \varepsilon$.

e.g. $\varepsilon = 11$



We regard this graph as a subspace of $\mathbb{R}^5 = \mathbb{E}^5$ by identifying

$H = (1,0,0,0,0) = e_1$

$M = (0,1,0,0,0) = e_2$

$R = (0,0,1,0,0) = e_3$

$C = (0,0,0,1,0) = e_4$

$W = (0,0,0,0,1) = e_5$

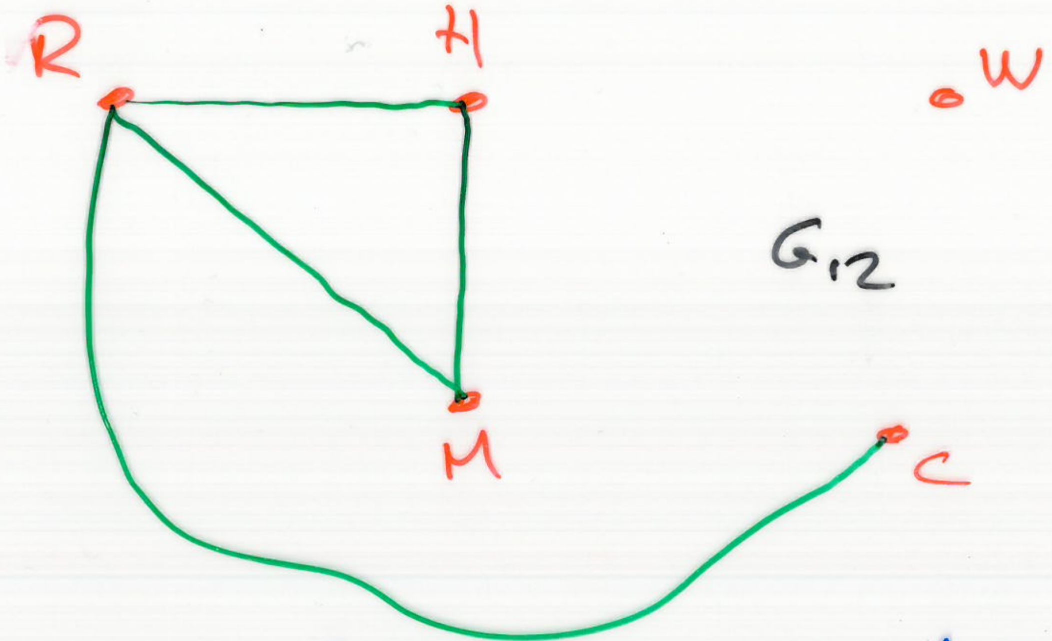
The graph G_{11} can be thought of as a subspace of $\mathbb{E}^5$ with points e_1, e_2, e_3, e_4, e_5 and the line segments

$$e_1 e_2 , \quad e_2 e_3 , \quad e_1 e_3 .$$

The graph G_{11} has three connected components:

$$X_W , \quad X_C , \quad X_{RHM}$$

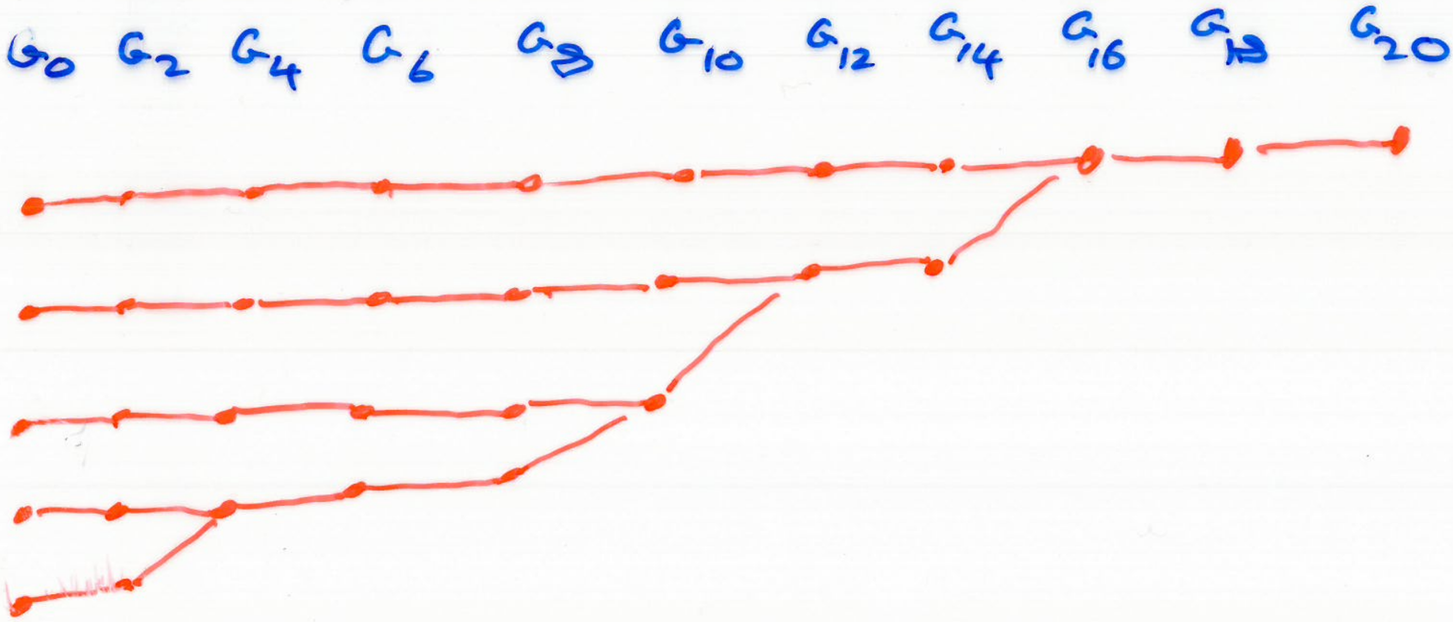
Now let's look at G_{12}:



G_{12}

So G_{12} has two connected components.

A dendrogram summarizes the inclusions of connected components.



Continuity

Defn Let X, Y be topological spaces.

A function
$$f : X \longrightarrow Y$$
is continuous if the inverse image of any open set in Y is an open set in X.

in other words, if $U \subseteq Y$ is open in Y then

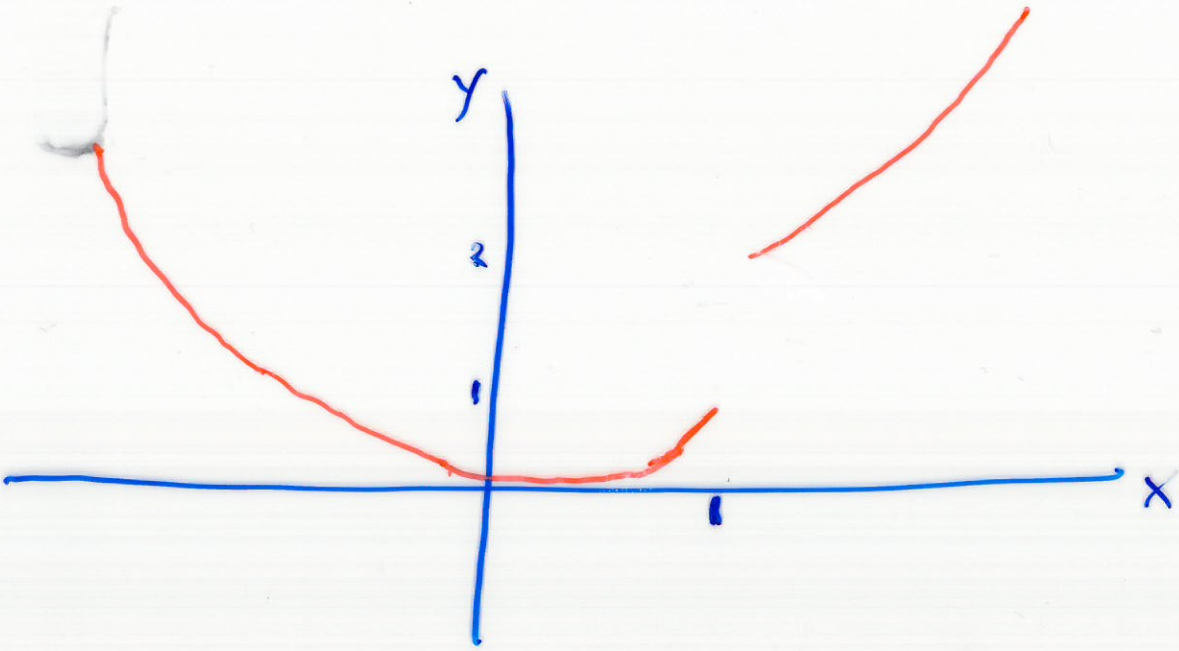
$$f^{-1}(U) = \{ x \in X : f(x) \in U \}$$

is open in X.

<u>Example</u> $X = \mathbb{E}^1$, $Y = \mathbb{E}^1$

Consider $f : \mathbb{E}^1 \to \mathbb{E}^1$ given by

$$f(x) = \begin{cases} x^2 , & x \leq 1 \\ x^2+1 , & x > 1 . \end{cases}$$



Consider the open set

$U = (-1, 2) \subseteq \mathbb{E}^1 = Y$.

The pre-image

$$f^{-1}(U) = (-\sqrt{2}, 1]$$

is <u>not</u> open in X.

Hence f is not continuous.