The sphere

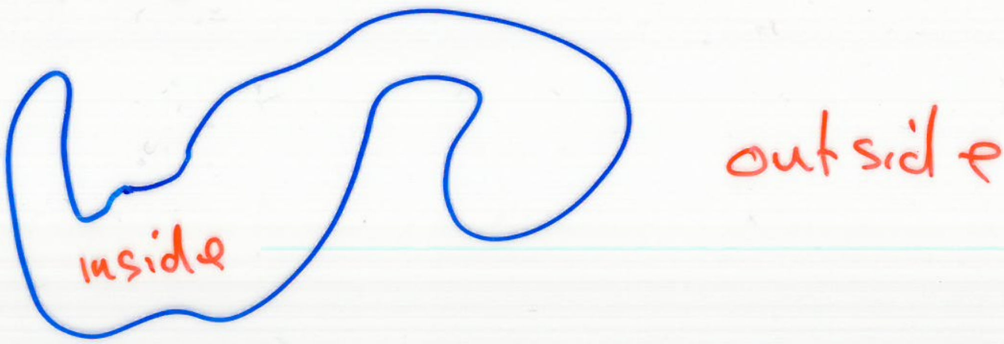
$$S^2 = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2=1\}$$

is a more precise notion than the "surface of Mars".

Our explanation of the Euler characteristic formula

$$\chi(S^2) = V - E + F = 2$$

used the fact that any loop on the sphere, with no self intersections

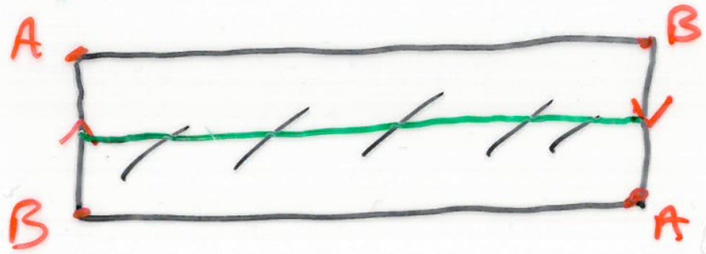


has an inside and an outside.

i.e. Any such loop cuts S^2 into two regions.

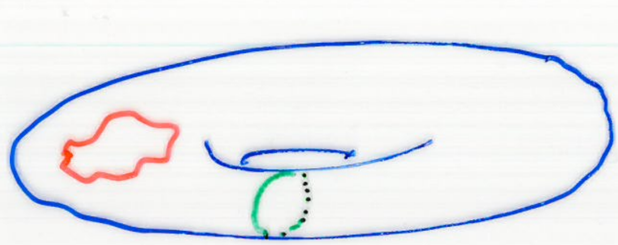
Question: Is this fact "obvious"?

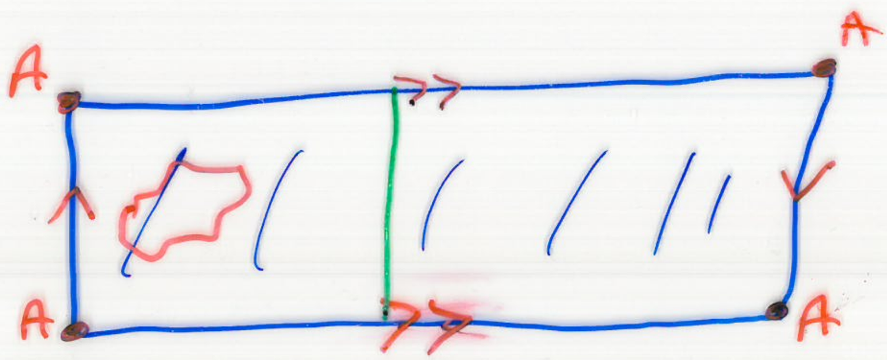
Example Consider the Möbius strip



Draw a loop around the centre of the strip, with paper, pen and scissors, check that the green loop does <u>not</u> cut the Möbius strip into two regions.

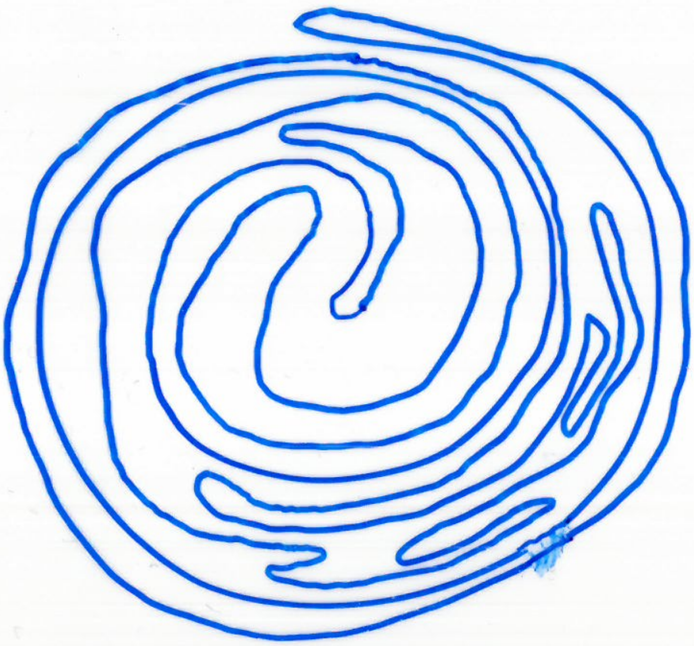
Example Consider a torus





Again, the green loop does **not** cut the torus into two regions.

Example Is it obvious that the following loop in $\mathbb{R}^2$ cuts the plane into two pieces?



Topology offers a precise language and a collection of techniques for studying such problems.

Let $S^1 = \{(x,y) \in \mathbb{R}^2 : x^2+y^2=1\}$.

So S^1 is our notion for a circle.

The MA342 module will initially aim at helping us understand the statement of the following result, and also its proof given in Armstrongs book.

<u>Jordan</u> <u>Curve</u> <u>Theorem</u>

Let $\alpha : S^1 \longrightarrow \mathbb{R}^2$ be any injective <u>continuous</u> function. Let $J \subseteq \mathbb{R}^2$ be the image of α. Then $\mathbb{R}^2 \setminus J$ has two <u>connected components</u>, both of which have <u>frontier</u> J.

Aim for next few lectures:

1) Explain above underlined terms

3) Give an <u>outline</u> explanation of the above theorem.

2) Give some weird examples that suggest the theorem is not so obvious.

Definition (Riesz [1909], Hausdorff [1914])

A topological space consists of a set X and a collection T of subsets of X which we call open. The following axioms must hold:

T1) The union of any collection of open sets is open.

T2) The intersection of any finite collection of open sets is open.

T3) Both ϕ and X are open.

Example 1 $X = \{1, 2, 3, 4\}$

$$T = \{\phi, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,3,4\}\}$$

This is a topological space.