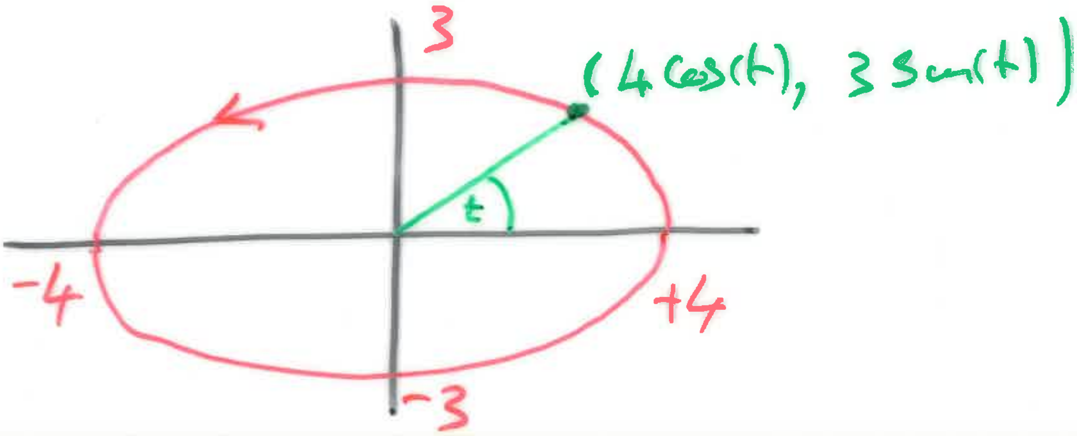
First test: Wed 9 october.

(Material based on lectures up to 30th september.)

Sol^n

$$\text{work} = \int_S (3x - 4y)\,dx + (4x + 2y)\,dy$$



$$x = 4\cos t$$
$$y = 3\sin t$$

$$dx = -4\sin t\,dt$$
$$dy = 3\cos t\,dt$$

work =

$$\int_0^{2\pi} -\big(3(4\cos t) - 4(3\sin t)\big)4\sin t\,dt + \big(4(4\cos t) + 2(3\sin t)\big)3\cos t\,dt$$

$$= \dots$$

$$= \int_0^{2\pi} 48 - 30(\sin t)(\cos t) \quad dt$$

$$= 48t - 15\sin^2 t \Big|_0^{2\pi}$$

$$= 96\pi .$$

Stokes' formula

$$\boxed{\int_{\partial S} \omega = \int_S d\omega}$$

when $\omega = f(x_1, x_2, \cdots, x_n)$ is a 0-form, and when S is a 1-dimensional, oriented, connected region:

- left-hand side makes sense to us.
- for the right-hand side we need to give a meaning to $d\omega$.

We call $d\omega$ the total derivative (also called the exterior derivative)

To define $d\omega$ we need:

Partial Derivatives

Given a 0-form

$$\omega = f(x, y, z)$$

we denote by

$$\frac{\partial f}{\partial x}$$

the 0-form obtained by regarding y and z as constants, and differentiating with respect to x.

We call $\frac{\partial f}{\partial x}$ the partial derivative of f with respect to x.

Example Consider

$$\omega = f(x, y, z) = \sqrt{1 - (x^2 + y^2 + z^2)}$$

defined on

$S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1\}.$

Calculate $\dfrac{\partial f}{\partial x}$.

<u>Sol^n</u> $f(x,y,z) = \left(1-(x^2+y^2+z^2)\right)^{\frac{1}{2}}$

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left(1-(x^2+y^2+z^2)\right)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x}{\sqrt{1-(x^2+y^2+z^2)}}.$$

Similarly

$$\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{1-(x^2+y^2+z^2)}}$$

$$\frac{\partial f}{\partial z} = \frac{-z}{\sqrt{1-(x^2+y^2+z^2)}}$$

<u>Notation</u>

We often write f_x in place of $\dfrac{\partial f}{\partial x}$.

Sol^n

$$dw = f_x\,dx + f_y\,dy + f_z\,dz$$

$$= \frac{-x}{\sqrt{1-(x^2+y^2+z^2)}}\,dx$$

$$- \frac{y}{\sqrt{1-(x^2+y^2+z^2)}}\,dy$$

$$- \frac{z}{\sqrt{1-(x^2+y^2+z^2)}}\,dz.$$

Total derivatives

Given a 0-form

$$w = f(x, y, z)$$

we define the 1-form

$$\boxed{dw = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz}$$

we call dw the total derivative of the 0-form w.

Example Find the total derivative of the 0-form

$$w = \sqrt{1 - (x^2 + y^2 + z^2)}$$

on

$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$