Hilbert's axioms

https://en.wikipedia.org/wiki/Hilbert%27s_axioms

Teaching Geometry at Second Level

Modern treatments of Euclidean Geometry are based on the axioms developed by David Hilbert in 1899.

Hilbert

- Did away with arguments based on spatial intuition and the construction of possibly misleading diagrams;
- Eliminated "superposition" arguments by adding new axioms on congruence;
- Added axioms to spell out the role of some concepts that were implicit in Euclid, e.g., "betweenness";
- Added axioms that spell out the modern understanding of the line as the set of all real numbers, both rational and irrational.

Geometry in Project Maths

The second level Geometry syllabus is motivated by the book "Geometry with Trigonometry", by P. D. Barry (2001).

The Geometry syllabus for Project Maths was written by Tony O'Farrell (Maynooth University). You can read it here:

Leaving Certificate Maths

The Undefined Terms

- angle,
- degree,
- length,
- line,
- plane,
- point,
- ray,
- real number,
- set.

The Axioms

Axiom 1 (The Two Points Axiom)

There is exactly one line through any two given points.
(We denote the line through A and B by AB.)

Axiom 2 (The Ruler Axiom)

The distance between points has the following properties:

1. The distance $|AB|$ is never negative.
2. $|AB| = |BA|$.
3. If C lies on AB, between A and B, then $|AC| + |CB| = |AB|$.
4. (Marking off a distance) Given any ray from A, and given any real number $k \geqslant 0$, there is a unique point B on the ray whose distance from A is k.

Axiom 3 (The Protractor Axiom)

The number of degrees in an angle (also known as its degree-measure) is always a number between $0°$ and $360°$. The number of degrees of an ordinary angle is less than $180°$. It has these properties:

1. A straight angle has $180°$.
2. Given a ray $[AB$, and a number d between 0 and 180, there is exactly one ray from A on each side of the line AB that makes an (ordinary) angle having d degrees with the ray $[AB$.
3. If D is a point inside an angle $\angle BAC$, then

$$\angle BAC = \angle BAD + \angle DAC.$$

Axiom 4 (SAS + ASA + SSS)

If one of the following holds, then the triangles ΔABC and $\Delta A'B'C'$ are congruent:

1. $|AB| = |A'B'|$, $|AC| = |A'C'|$ and $|\angle A| = |\angle A'|$.
2. $|BC| = |B'C'|$, $|\angle B| = |\angle B'|$ and $|\angle C| = |\angle C'|$.
3. $|AB| = |A'B'|$, $|BC| = |B'C'|$ and $|CA| = |C'A'|$.

Axiom 5 (The Parallel Axiom)

Given any line ℓ and a point P, there is exactly one line through P that is parallel to ℓ.

Theorem (Vertically-Opposite Angles)

Vertically opposite angles are equal in measure.

Theorem (Isosceles Triangles)

1. *In an isosceles triangle the angles opposite the equal sides are equal.*
2. *Conversely, if two angles are equal, then the triangle is Isosceles.*

Theorem (Alternate Angles)

Suppose that A *and* D *are on opposite sides of the line* BC.

1. *If* $|\angle \mathsf{ABC}| = |\angle \mathsf{BCD}|$ *then* $\mathsf{AB} \| \mathsf{CD}$. *In other words, if a transversal makes equal alternate angles on two lines, then the lines are parallel.*
2. *Conversely, if* $\mathsf{AB} \| \mathsf{CD}$, *then* $|\angle \mathsf{ABC}| = |\angle \mathsf{BCD}|$. *In other words, if the two lines are parallel, then any transversal will make equal alternate angles with them.*