

MA334 Geometry

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Assessment

- Continuous assessment 30%
- Exam 70%

Course website

<http://www.maths.nuigalway.ie/~mstudies/ma334/index.shtml>

- Lectures
- Problem sheets
- Other materials

Tutorials

- Attend 1 tutorial p/w (times and venues TBC)
- Problem sheets
- Class tests

Recommended reading

Geometry by Roger Fenn. Springer Undergraduate Mathematics Series.

Any other geometry textbook in the library!

Outline

- Axiom systems
- Hyperbolic Geometry
- Projective Geometry
- Spherical Geometry
- Euclidean Geometry

The axiomatic method

Introduced by Euclid in ‘Elements’, approx 300 BC

- **Undefined terms:** Things that we accept as basic terms not requiring definition.
- **Axioms (postulates):** Statements that we accept without proof.
- **Rules of reasoning:** Agreed procedures that we can use to say that one statement follows from another.

Theorems: Statements that follow from the axioms by means of a sequence of steps (the **proof**) using the rules of reasoning.

Undefined terms

- ① Point
- ② Line
- ③ Plane
- ④ Between ('The point A is between the points B and C')
- ⑤ Lies on ('The point A lies on the line L')
- ⑥ Congruent ('Two line segments are congruent', 'Two angles are congruent')

Euclid's five axioms

- 1 For every point P and for every point Q not equal to P there exists a unique line ℓ that passes through P and Q .
- 2 For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE . (Another version: Any straight line segment can be extended indefinitely in a straight line.)
- 3 For every point O and every point A not equal to O there exists a circle with center O and radius OA .
- 4 All right angles are congruent to each other.
- 5 For every line ℓ and for every point P that does not lie on ℓ there exists a unique line m through P that is parallel to ℓ . (Original version: If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.)

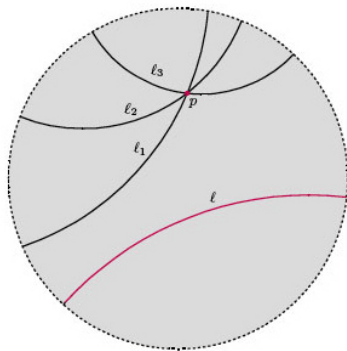
Euclid's fifth axiom (The 'Parallel Postulate')

- ⑤ For every line ℓ and for every point P that does not lie on ℓ there exists a unique line m through P that is parallel to ℓ .

This axiom is very different in nature from the others. The first four axioms can be 'verified' by diagrams. The fifth axiom cannot, although it may seem intuitively obvious. For many centuries, mathematicians tried to prove the Parallel Postulate from the other axioms. The discovery in the 19th century of 'Non-Euclidean Geometries' showed that this is impossible.

Hyperbolic geometry

Suppose we reject the parallel postulate, i.e., suppose we can have more than one line through a point p parallel to the line ℓ .



Projective geometry

In projective geometry there are *no* parallel lines. Lines either intersect in a point in the ‘usual’ sense, or they meet at a ‘point at infinity’ or ‘on the horizon’.



Spherical geometry

We live on the surface of a sphere. Straight lines on the earth are arcs of great circles.

