

lec [23]

In the last example
we found the FULL Laurent
Series

but we only needed ONE
coefficient i.e. of $\frac{1}{(z-z_0)^1}$ (b_1)

If the pole has order 1

$$f(z) = \frac{b_1}{z-z_0} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \dots$$

$$\Rightarrow f(z)(z-z_0) = b_1 + a_0(z-z_0) + a_1(z-z_0)^2 + \dots$$

$$\Rightarrow \boxed{\lim_{z \rightarrow z_0} [f(z)(z-z_0)] = b_1}$$

Ex Find the residue (i.e. b_1)

of $f(z) = \frac{e^z}{z}$ at $z=0$.

If the pole is order 1, then

$$b_1 = \lim_{z \rightarrow 0} f(z) (z-0)$$

$$= \lim_{z \rightarrow 0} \frac{e^z}{z} \cdot z = \lim_{z \rightarrow 0} e^z = 1.$$

(check this by writing out the full Laurent series). └

If the pole has order 2

$$f(z) = \frac{b_2}{(z-z_0)^2} + \frac{b_1}{z-z_0} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + \dots$$

$$\Rightarrow f(z)(z-z_0)^2 = b_2 + b_1(z-z_0) + a_0(z-z_0)^2 + \dots$$

$$\left[f(z)(z-z_0)^2 \right]' = b_1 + 2a_0(z-z_0) + 3a_1(z-z_0)^2 + \dots$$

$$\lim_{z \rightarrow z_0} [f(z)(z-z_0)^2]' = b_1$$

Ex

$$f(z) = \frac{50z}{(z+4)(z-1)^2}$$

Find: Res $f(z)$
 $z=1$

$$\lim_{z \rightarrow 1} [f(z)(z-1)^2]'$$

$$= \lim_{z \rightarrow 1} \left[\frac{50z}{(z+4)(z-1)^2} \cdot (z-1)^2 \right]'$$

$$= \lim_{z \rightarrow 1} \frac{d}{dz} \left(\frac{50z}{z+4} \right)$$

$$= \frac{(z+4)(50) - 50z}{(z+4)^2} \Big|_{z=1}$$

$$= \frac{(5)(50) - 50}{25} = 8$$