

Lec 21

Ex

Expand

$$\frac{3}{(z-2)(z+1)} = f(z)$$

in a Laurent series valid

in (a)  $0 < |z+1| < 3$

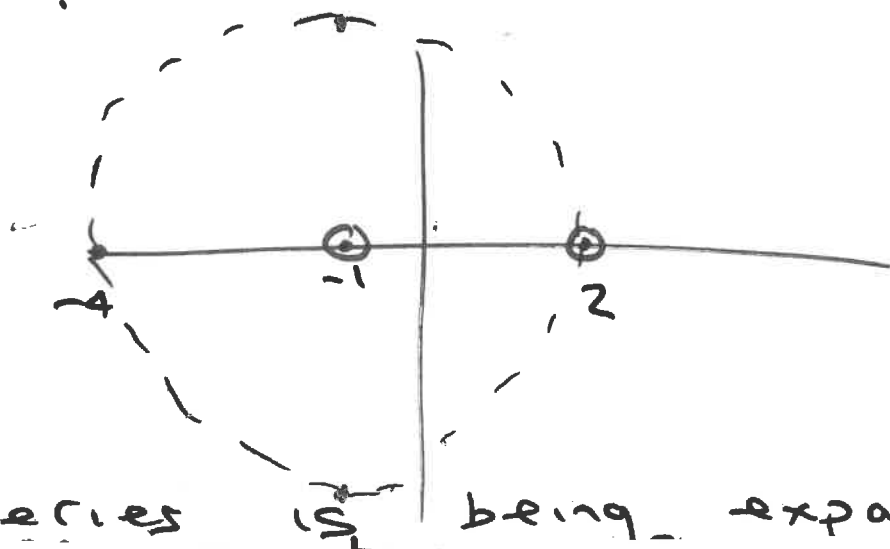
(b)  $|z+1| > 3$ .

Sol

First note  $f(z)$

is singular at  $z=2$  and

$z=-1$ .



The series is being expanded away away from these two singular points.

Note also  
 you need to expand  $f(z)$   
 around  $z = -1$   
 i.e. in powers of  $z+1$ .

$$(a) \quad f(z) = \frac{3}{(z-2)(z+1)}$$

$$= \frac{3}{z+1} \left( \frac{1}{z-2} \right)$$

$$= \frac{3}{z+1} \left[ \frac{1}{(z+1)-3} \right]$$

$$= -\frac{3}{z+1} \left[ \frac{1}{3-(z+1)} \right]$$

$$= -\frac{3}{z+1} \frac{1}{3} \left[ \frac{1}{1-\left(\frac{z+1}{3}\right)} \right]$$

Note:  $\frac{1}{1-x}$

$$= \frac{-1}{z+1} \left[ 1 + \left(\frac{z+1}{3}\right) + \left(\frac{z+1}{3}\right)^2 + \dots \right]$$

when  $\left|\frac{z+1}{3}\right| < 1$

or  $|z+1| < 3$

as  $z \neq -1$ , we have  $0 < |z+1| < 3$

$$\text{So } f(z) = -\frac{1}{z+1} - \frac{1}{3} - \frac{z+1}{3^2} - \frac{(z+1)^2}{3^3} \dots$$

$$(b) f(z) = \frac{3}{(z-2)(z+1)}$$

$$= \frac{3}{z+1} \frac{1}{z-2}$$

$$= \frac{3}{z+1} \frac{1}{(z+1)-3}$$

$$= \frac{3}{(z+1)^2} \left[ \frac{1}{1 - \frac{3}{z+1}} \right] \quad \text{NOTE } \frac{1}{1-x}$$

$$= \frac{3}{(z+1)^2} \left[ 1 + \frac{3}{z+1} + \left(\frac{3}{z+1}\right)^2 + \dots \right]$$

$$\text{where } \left| \frac{3}{z+1} \right| < 1$$

$$\text{or } 3 < |z+1|$$

$$\text{or } |z+1| > 3$$

$$\text{So } f(z) = \frac{3}{(z+1)^2} + \frac{3^2}{(z+1)^3} + \frac{3^3}{(z+1)^4} + \dots$$