

Note 2 ^{Lec 4}

Just like real functions, the above function is not defined when the denominator equals zero.

In this case, the values of z when

$$(z^2 - 2iz)^2 = 0$$

$$\text{or } z^2 - 2iz = 0$$

$$\text{or } z(z - 2i) = 0$$

of when $z = 0$ and $z = 2i$
($0 + 0i$) ($0 + 2i$)

Note 3

It is possible to find the derivative of these functions

away from points
when the denominator
equals zero.

The same differentiation
rules that apply to
real functions
apply to functions of a
complex variable.

Ex

Differentiate $f(z) = \frac{ie^{iz}}{(z^2 - 2iz)^2}$

This is a quotient

Recall $d\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

$$\frac{du}{dz} = i [ie^{iz}] = i^2 e^{iz} = -e^{iz}$$

$$\frac{dv}{dz} = 2(z^2 - 2iz)(2z - 2i)$$

So

$$\frac{df(z)}{dz} = \frac{v \frac{du}{dz} - u \frac{dv}{dz}}{v^2}$$

with $u = ie^{iz}$ $v = (z^2 - 2iz)^2$

$$f'(z) = \frac{(z^2 - 2iz)^2 (-e^{iz}) - ie^{iz} [2(z^2 - 2iz)(2z - 2i)]}{(z^2 - 2iz)^4}$$

$$= \frac{[-(z^2 - 2iz) - 2i(z^2 - 2iz)] e^{iz}}{(z^2 - 2iz)^3}$$

$$= \frac{-[(z^2 - 2iz) + 2i(z^2 - 2iz)] e^{iz}}{(z^2 - 2iz)^3}$$

$$= \frac{-[z^2 - 2iz + 4iz - 4i^2] e^{iz}}{(z^2 - 2iz)^3}$$

$$= \frac{-[z^2 + 2iz + 4] e^{iz}}{(z^2 - 2iz)^3}$$

Ex Given $f(z) =$

$$f(z) = \cos(z^2) + (z^3 + 2iz)^3$$

(a) find the finite z if any where $f(z)$ is not defined

(b) differentiate $f(z)$.

(a) $f(z)$ is defined at all finite z .

(There are no $\frac{1}{0}$ scenarios)

$$\begin{aligned} (b) \quad \frac{df}{dz} &= [-\sin(z^2)] \cdot (2z) \\ &\quad + 3(z^3 + 2iz)^2 (3z^2 + 2i) \\ &= -2z \sin(z^2) + 3(z^3 + 2iz)^2 (3z^2 + 2i) \end{aligned}$$