Lec 18

Cauchy's Integral Formula

Let $f(z)$ be analytic in a simply connected domain D,

then for any point z_0 in D and any simple closed path C in D that encloses z_0

$$\oint_C \frac{f(z)}{z - z_0}\, dz = 2\pi i\, f(z_0)$$

<u>Ex</u>

$$\oint_C \frac{e^z}{z-2}\, dz$$

where C is

(a) any contour that encloses $z_0 = 2$

(b) any contour that does not contain $z_0 = 2$

Sol

(a) $= 2\pi i e^{2} \doteqdot (0) + (46.43) i$

(b) By Cauchy's integral theorem $= 0$.

Ex

$$\oint_C \frac{z^3-6}{2z-i}\, dz$$

$$= \oint_C \frac{(z^3-6)/2}{z - i/2}\, dz$$

$$= 2\pi i (z^3-6)/2 \Big|_{z=i/2} = \frac{\pi}{8} - 6\pi i$$

if C contains $z_0 = i/2$

otherwise $= 0$.

Exercise — 4 separate parts.

Integrate $\dfrac{z^2+1}{z^2-1}$ around a, b, c, D