$\boxed{\text{Lec } 15}$

Complex Integration

Recall, definite integrals

e.g $\displaystyle\int_1^2 x^2 \, dx$

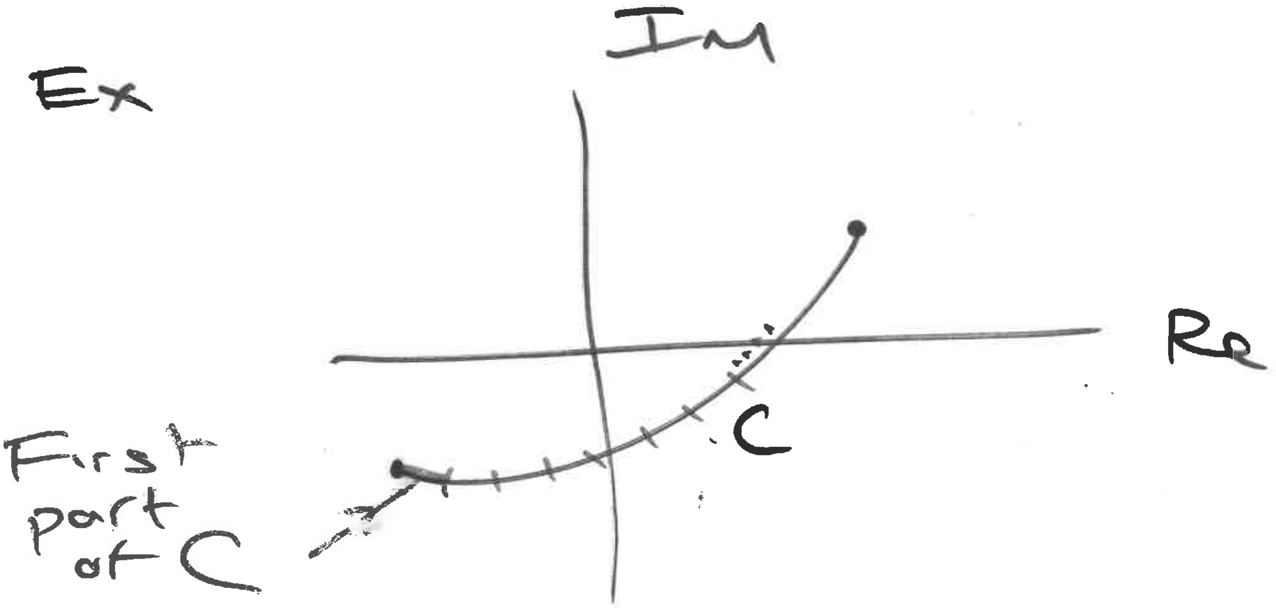
Complex definite integrals (complex line integrals) are written as:

$$\int_C f(z)\, dz$$

Here $f(z)$ – the integrand is integrated over the curve C over the complex plane.

What does this mean?

Ex



C is broken into n parts

The function $f(z)$ is valued somewhere in the first part.

This is multiplied by the length of the first part of C giving $f(P_1)\,\Delta z_1$

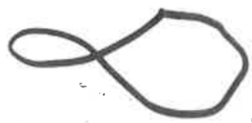
This is done for all the other parts of C and these results are added

$$S_n = \sum_{i=1}^{n} f(P_i)\,\Delta z_i$$

The integral is obtained by letting $n \to \infty$

So $$\int_C f(z)\,dz = \lim_{n\to\infty} \sum_{i=1}^{n} f(P_i)\,\Delta z$$

C is called <u>the path of integration</u>

C is a closed path if the end point is the same as the start point e.g  or 

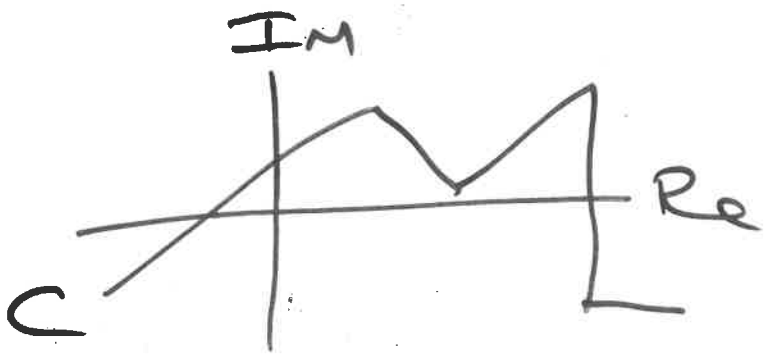
For a closed path C, we have $\oint_C f(z)d$

General assumption:

All paths of integration are assumed to be piecewise smooth i.e.

they consist of finitely many smooth curves joined end to end

e.g



We have two integration methods.

First some definitions.

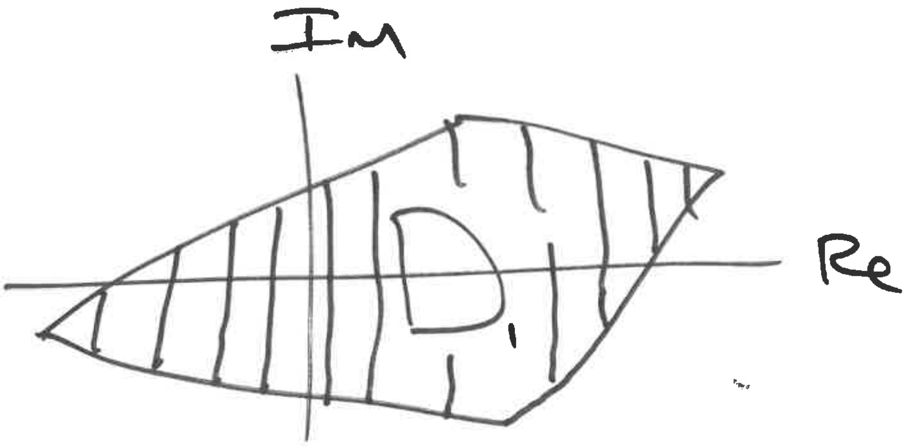
A simple closed curve is a closed curve in the complex plane with no self-intersection.

e.g  or  are simple closed curves.

 is not a simple closed curve.

A region D in the complex plane is simply connected if every simple closed curve in D encloses only points of D.

Ex 1



D_1 is simply connected because no matter where you draw a simple closed curve C in D_1 the interior of C_1 consists of all points of D_1

Ex 2

D_2 is <u>not</u> simply connected. This is because the interior of C_2 does not <u>fully</u> consist of points in D_2

(A simply connected region in the complex plane <u>does</u> not have any holes.