

# MA208 Quantitative Techniques for Business

## Lecture 8: Probability ctd.

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Today we will talk about

- Random variables,
- Discrete / continuous random variables,
- Probability distributions,
- Binomial Probability Distribution.

# Random Variables

A **random variable** is a numerical variable whose value is determined by an underlying random experiment.

## Example

I toss a coin five times. This is a random experiment and the sample space can be written as

$$S = \{TTTTT, TTTTH, \dots, HHHHH\}.$$

Note that here the sample space has  $2^5 = 32$  elements. Suppose that in this experiment, we are interested in the number of heads. We can define a random variable  $X$  whose value is the number of observed heads. The value of  $X$  will be one of **0, 1, 2, 3, 4** or **5** depending on the outcome of the random experiment.

# Probability Distribution

A random variable has a **probability distribution**, which specifies the probability that its value falls in any given interval.

## Example

Random variable	Probability
10 - 19	0.04
20 - 29	0.08
30 - 39	0.14
40 - 49	0.20
50 - 59	0.32
60 - 69	0.16
70 - 79	0.06

The random variable is the exam result received by a student.

Depending on the type of the experiment, the numerical value that the random variable takes can be classified as **discrete random variable** or **continuous random variable**.

- **Discrete random variable:** the set of assumed values is *countable*, e.g. numbers of players in a team, number of visits in a doctor's surgery;
- **Continuous random variable:** the set of assumed values is *uncountable*, e.g. amount of rainfall in a month, the time required to cycle to university.

## Examples

- (1) Two balls are drawn in random in a succession without replacement from an urn containing four red balls and six black balls. Find the possibilities of all possible outcomes.

## Solution

Let  $X$  denote the number of red balls in the outcome.

Possible outcomes	RR	RB	BR	BB
$x_i$	2	1	1	0

Here  $x_1 = 2$ ,  $x_2 = 1$ ,  $x_3 = 1$ ,  $x_4 = 0$

## Solution

The probability of getting 2 red balls when we draw the balls one at a time is

- Probability of 1st ball being red =  $\frac{4}{10}$
- " " 2nd " " " =  $\frac{3}{9}$

$$\text{So } P(X_1) = \frac{4}{10} \cdot \frac{6}{9} = \frac{4}{15}$$

$$\text{Similarly, we get } P(X_2) = \frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$$

$$P(X_3) = \frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$

$$P(X_4) = \frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$$

Check: If we have found all probabilities, they should add up to 1.

$$\frac{2}{15} + \frac{4}{15} + \frac{4}{15} + \frac{1}{3} = \frac{15}{15} = 1 \checkmark$$

## Examples

- (2) A jar coffee is picked at random from a filling process in which an automatic machine is filling coffee jars each with 1 kg of coffee. Due to some faults in the automatic process, the weight of a jar could vary from jar to jar between 0.9 kg and 1.05 kg. Let  $X$  denote the weight of a jar of coffee selected. What is the range of  $X$ ?

## Solution

Possible outcomes  $0.9 \leq x \leq 1.05$

(continuous variable)

# Probability Distributions

There are some specific distributions that are used over and over in practice, thus they have been given special names. There is a random experiment behind each of these distributions. Since these random experiments model a lot of real life phenomenon, these special distributions are used frequently in different applications. That's why they have been given a name and we devote some time to study them.

Do not get intimidated by the complicated looking formulas, look at each distribution as a practice problem on discrete random variables.

# Binomial Probability Distribution

## Example

Suppose that I have a coin with  $P(H) = p = 0.5$ . I toss the coin  $n = 30$  times and define  $X$  to be the total number of heads that I observe. Then  $X$  is **binomial** with parameters  $n$  and  $p$ . The range of  $X$  in this case is  $\{0, 1, 2, \dots, 30\}$ . What is the probability to get 17 heads?

We can use the formula for **Binomial Probability Distribution** (see next slide):

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}.$$

## Example

$$P(X = 17) = \binom{30}{17} \cdot (0.5)^{17} \cdot (1 - 0.5)^{(30-17)} \approx 0.111535052$$

# Binomial Probability Distribution

The probability distribution of the random variable is called a **binomial distribution** if

- the experiments consist of a fixed number of  $n$  trials,
- the probability of success (denoted by  $p$ ) remains constant,
- all trials are independent,
- each trial is either a success or failure (hence the name binomial).

The binomial distribution can be described by

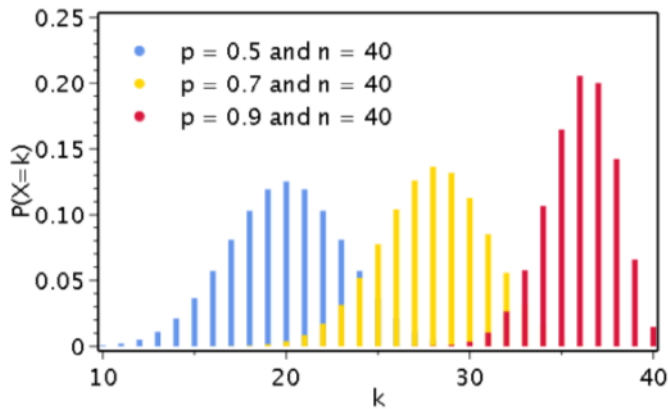
## Binomial Probability Distribution

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

where  $n$  is the number of trials,  $x \in \{0, 1, 2, 3, \dots, n\}$  is the discrete variable and  $p$  is the probability of success in a single trial.

# Binomial Probability Distribution

## Example



# Binomial Probability Distribution

## Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

## Solution

- independent events
  - success / failure
- } Binomial distribution

Sln.  $X := \#$  of persons who buy ice cream  
 $n = 6$  trials, we want 6 success

$$P(X=4) = \binom{6}{4} (0.3)^4 (1-0.3)^{6-4}$$
$$= \dots = 0.06$$

# Binomial Probability Distribution

## Example

A die is tossed three times. What is the probability of

- (i) no fives?
- (ii) one five?
- (iii) three fives?

## Solution

Only 2 outcomes (getting a 5 or not), so Binomial dist.

Sol.  $n=3$   
Let  $x$  = number of fives appearing,  $p = \frac{1}{6}$

(i)  $x=0$   
$$P(x=0) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 = \frac{125}{216} = 0.5787$$

(ii)  $x=1$   
$$P(x=1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = \frac{75}{216} = 0.34722$$

(iii)  $x=3$   
$$P(x=3) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 = \frac{1}{216} = 4.6296 \times 10^{-3}$$

# Binomial Probability Distribution

## Example

A blindfolded marksman finds that on the average he hits the target four times out of five. If he fires four shots, what is the probability of

- (i) more than two hits?
- (ii) at least three misses?

## Solution

*Again hit or miss  $\rightarrow$  2 possible outcomes  $\rightarrow$  Binomial*

Here  $n=4$ ,  $p=\frac{4}{5}=0.8$ ,  $x$  = number of hits

$$(a) \quad P(x > 2) = P(x=3) + P(x=4) \\ = \binom{4}{3} (0.8)^3 (1-0.8)^1 + \binom{4}{4} (0.8)^4 (0.2)^0 = \dots = 0.8192$$

$$(b) \quad 3 \text{ misses} \hat{=} 1 \text{ hit, } 4 \text{ misses} \hat{=} 0 \text{ hits. Looking for } P(x \leq 1) \\ P(x \leq 1) = P(x=1) + P(x=0) = \dots = 0.0272$$















