MA208 Quantitative Techniques for Business

Lecture 20: Linear Inequalities and Linear Programming ctd.

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Linear programming is a mathematical optimisation technique, where optimisation means maximizing or minimizing a linear equation subject to some constraints.

Linear programming: the corner point method

Example

A factory makes two types of machine, *A* and *B*. The table below details resource requirements for one of each type of machine.

Resource	Machine *A*	Machine *B*	Monthly availability
Energy	100 kWh	200 kWh	20,000 kWh
Steel	60 kg	80 kg	10,000 kg
Labour	2.5 h	2 h	400 h

The company makes a profit of €48 per machine *A* and €38 per machine *B* produced. How many of each type of machine should the company produce to maximize profit?

Solution

Let x be the number of machine A and y be the number of machine B.
*The total profit can be described by the **objective function**:*

$$p = 48x + 38y$$

*We must maximise this function subject to the following **constraints**:*

$$\begin{array}{rcrcr} 100x & + & 200y & \leq & 20,000 \\ 60x & + & 80y & \leq & 10,000 \\ 2.5x & + & 2y & \leq & 400 \end{array}$$

We also assume that $x \geq 0$ and $y \geq 0$.

Corner Point Method

To solve a linear programming problem with two decision variables

1. Graph the region satisfying all of the inequalities. This is called region of feasible solutions.
2. Find the co-ordinates of all of the corner points of the region.
3. Substitute each into the function to be maximised.
4. One of these is the answer!

Note: To solve linear programming problems with more than two variables, more advanced methods are required, for example the “Simplex Method”.

Solution (ctd.)

Recall: we must

maximize

$$38x + 38y$$

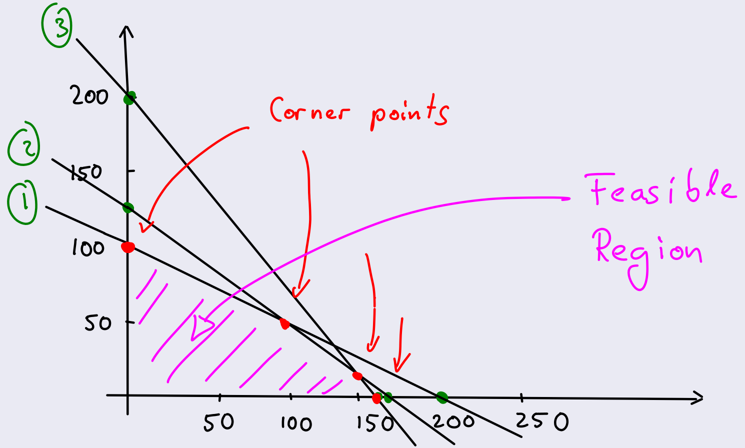
subject to

$$\begin{array}{rrcrcr} (1) & 100x & + & 200y & \leq & 20,000 \\ (2) & 60x & + & 80y & \leq & 10,000 \\ (3) & 2.5x & + & 2y & \leq & 400 \end{array}$$

First, find the feasible region. To do that, draw graphs of the three inequalities.
The inequalities $x \geq 0$ and $y \geq 0$ indicate that the solution will lie in the first quadrant.

Linear Programming

Solution (ctd.)



Solution (ctd.)

Now find the coordinates of the four corner points.

Fact: *The optimal solution must occur at one of the corner points.*

We know already two of the corner points, namely those on the x- *and* y *axis:*

$$(0, 100)\ ,\ (160, 0).$$

The other two corner points are the intersection of lines (1) and (2) and the intersection of lines (2) and (3). Thus, we have to find those. (Board work!)

Linear Programming

Solution (ctd.)

The intersection point of lines (1) and (2) is $(100, 50)$*,*
and the intersection point of lines (2) and (3) is $(150, 12.5)$*.*

So the ***corner points*** *are*

$$(0, 100), (160, 0), (100, 50), (150, 12.5).$$

To find out which of those is the solution, we need to substitute each of them into the profit function

$$p = 48x + 38y$$

Solution (ctd.)

$$p = 48x + 38y$$

- $(0, 100)$ $\quad p = 48(0) + 38(100) = 3,800$
- $(160, 0)$ $\quad p = 48(160) + 38(0) = 7,680$
- $(100, 50)$ $\quad p = 48(0) + 38(100) = 3,800$
- $(150, 12.5)$ $\quad p = 48(0) + 38(100) = 3,800$

So the maximum profit is €$7,680$ *which happens when the factory makes* 160 *machines of type* A *and* 0 *machines of type* B.

Example

Example (from 2015 exam paper - rephrased)

An oil company wish to inject a fracking fluid into the ground of an island and can use two types of chemicals, Bromine and DBNPA. Both types have an impact on the environment and therefore there are limits on the impact type.

Impact type	Bromine	DBNPA	Limit for permitted damage
Drinking water	40 kTpU	20 kTpU	1200 kilotons
Air pollution	30 kTpU	30 kTpU	1800 kilotons
Surface contamination	25 kTpU	50 kTpU	1500 kilotons

(kTpU = kilotons per unit)

Example (ctd.)

(a) Let x and y denote the number of units to be extracted with Bromine (resp. DBNPA). Write down three inequalities representing the constraints on x and y imposed by the above table.

(b) Sketch these inequalities and shade the region of feasible solutions.

(c) The company makes a profit of €13 million per unit of gas extracted with Bromine and of €7 million per unit extracted with DBNPA. How many units extracted with Bromine together with how many extracted with DBNPA do maximize the profits? And what is the maximum profit?