MA208 Quantitative Techniques for Business

Lecture 14: Mathematics of Finance ctd.

Dr Kirsten Pfeiffer

School of Mathematics, Applied Mathematics and Statistics
NUI Galway

Lecture 14 - Outline

Today we will talk about

- Present value of an Annuity
- Amortization

Revision: Present value of an Annuity

Present value of an Annuity

In general the Present Value of an Ordinary Annuity is

$$PV = PMT \frac{(1 - (1 + i)^{-n})}{i}$$

where PV = present value of all payments, PMT = periodic payment, i = rate per period, and n = number of periods.

Present value of an Annuity

Example

What is the present value of an annuity that pays €200 per month for five years if money is worth 6% compounded monthly?

Solution

$$PMT = €\ 200, \quad i = \frac{0.06}{12} = 0.005, \quad n = (5)(12) = 60$$

$$PV = PMT\ \frac{1-(1+i)^{-n}}{i}$$

$$= 200\ \frac{1-(1.005)^{-60}}{0.005}$$

$$= €\ 10,345$$

Amortization

The present value formula for an ordinary annuity is also used to determine the distribution of loan repayments, as determined by an *amortization* schedule.

Example

Suppose you borrow €5000 from a bank to buy a car and agree to repay the loan in 36 equal monthly payments, including all interest due. If the bank charges 1% per month on the unpaid balance (i.e. 12% per year compounded monthly), how much should each payment be?

Amortization

Here, the bank has bought an annuity from you. So the question is: If the bank pays you $PV = €5000$ for an annuity, what are the periodic payments PMT?

Solution

We have $PV = 5000, i = 0.01, n = 36$.

$$\begin{aligned} PV &= PMT \frac{(1-(1+i)^{-n})}{i} \\ 5000 &= PMT \frac{(1-(1.01)^{-36})}{0.01} \\ PMT &= €166.07 \end{aligned}$$

Amortization

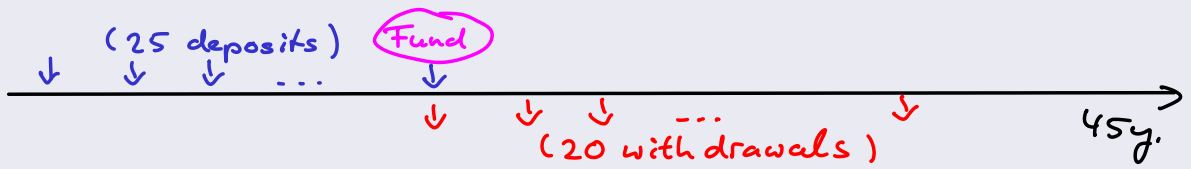
Example

You wish to purchase a retirement plan that earns 6.5% interest compunded annually. You will make annual deposits into this account for 25 years, and then make 20 equal withdrawals of €10,000 reducing the balance to zero. Compute

1. the value of the fund based on the withdrawals,
2. the amount of each deposit in order to maintain the fund,
3. the total interest earned over the entire 45 years.

Solution

It may help to sketch a time line:



Amortization

Solution

This is what you need to do:

1. Use the Present Value Formula to calculate the fund value.
   (answer: € 110,185.07)

2. Now use the Future Value Formula to calculate the amount of the deposits.
   (answer: € 1871.06)

3. Total interest = (total withdrawals) − (total deposits) = ?

Work through the steps yourself and compare the answers!
(Let me know if I made an error!)