MA208 Quantitative Techniques for Business

Lecture 11: Mathematics of Finance

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In this Section we will cover

- Simple Interest
- Compound Interest, Continuous Compound Interest
- Future Value of an Annuity, Sinking Funds
- Present Value of an Annuity, Amortization

Lecture 10 - Outline

Today we will talk about

- Simple Interest
- Compound Interest

Simple Interest

Simple interest is generally used only on short term loans or deposits. If you deposit or borrow a sum of money P (called the principal), a fee (called interest) is charged. This is a fee charged to you for borrowing money from a bank or a fee the bank pays you for the use of money you have deposited in a savings account. It's usually computed as a percentage, called the interest rate, of the principal over a given period of time. Simple interest is calculated using the following formula:

Simple Interest

$$I = Prt$$

where $I =$ interest, $P =$ principal, $r =$ simple interest rate (as a decimal) and $t =$ time in years.

Simple Interest

Example

The interest on a loan of €500 at 12% per annum for nine months is

$$
\begin{aligned}
I &= Prt \\
&= (500)(0.12)(0.75) \\
&= €45.
\end{aligned}
$$

Note that nine months as a fraction of one year is $\frac{9}{12} = 0.75$.

At the end of nine months the borrower must repay the principal plus the interest, in total €545.

Simple Interest

In general, if a principal P is borrowed at a rate r, then after t years the borrower will owe the lender an amount A that includes the principal P plus the interest I:

$$\begin{aligned} A &= P + I \\ &= P + Prt \\ &= P(1 + rt) \end{aligned}$$

Simple Interest Formula

$$A = P(1 + rt)$$

Note: We call A the future value, and the principal P is often called present value.

Example

Find the total amount due on a loan of €800 at 9% simple interest after 4 months.

Solution

$$A = P(1 + rt)$$

$$= 800\left(1 + (0.09)\left(\frac{4}{12}\right)\right)$$

$$= 800\left(1 + (0.09)\left(\frac{1}{3}\right)\right)$$

$$= €\ \underline{\underline{824}}$$

Simple Interest

Example

A family member has loaned you €1,000 with the understanding that the principal plus 4% p/a simple interest are to be repaid when you are able. How much would you owe if you repaid the loan after

(i) 2 years?

(ii) 10 years?

Solution

(i) $A = 1{,}000\,(1 + (0.04)(2))$
$= 1{,}000\,(1.08)$
$= 1{,}080$ [€]

(ii) $A = 1{,}000\,(1 + (0.04)(10))$
$= 1{,}000\,(1.4)$
$= 1{,}400$ [€]

Simple Interest

Example

Your current car loan will be repaid in nine months time. At that time you intend to purchase a newer model and expect that with the trade in value of the current vehicle you will require an additional €5000. Your local credit union just announced a new short term savings account with an interest rate of 10%. How much should you deposit now in order to have a total of €5000 after nine months?

Solution

$$A = P(1 + rt)$$

$$5{,}000 = P(1 + (0.1)(9/12))$$

$$= P(1{,}075)$$

$$P = \frac{5{,}000}{1{,}075}$$

$$= [€]\ 4{,}651.16$$

If at the end of a payment period the interest due is reinvested, then the interest as well as the principal will earn interest during the next payment period. Interest paid on interest reinvested is called compound interest.

Let's look at an example to obtain a formula that allows us to carry out calculations involving compound interest.

Example

You deposit €$1,000$ in a bank that pays 10% p/a compounded quarterly. How much will your deposit be worth in 1 year?

Compound Interest

Solution

Using the simple interest formula let's calculate the interest earned in each quarter. In the first quarter $P = €1,000$, $r = 0.1$ and $t = 0.25$. So the amount at the end of the first quarter is

$$\begin{aligned} A &= P(1 + rt) \\ &= 1,000(1 + (0.1)(0.25)) \\ &= 1,000(1 + 0.025) = €1,025. \end{aligned}$$

At the beginning of the second quarter the principal is $P = €1,025$. And the amount at the end of the second quarter is

$$\begin{aligned} A &= P(1 + rt) \\ &= 1,025(1 + (0.1)(0.25)) \\ &= 1,025(1 + 0.025) = €1,050.63. \end{aligned}$$

Solution ctd.

At the beginning of the third quarter the principal is $P = €1,050.63$. And the amount at the end of the third quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,050.63(1+(0.1)(0.25)) \\ &= 1,050.63(1+0.025) = €1,076.90. \end{aligned}$$

At the beginning of the fourth quarter the principal is $P = €1,076.90$. And the amount at the end of the fourth quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,076.90(1+(0.1)(0.25)) \\ &= 1,076.90(1+0.025) = €1,103.82. \end{aligned}$$

If we trace the calculations backwards through the last example we can see that the amount at the end of the fourth quarter was

$$
\begin{aligned}
€1,103.82 &= 1,076.90(1+0.025) \\
&= 1,050.63(1+0.025)(1+0.025) \\
&= 1,025(1+0.025)(1+0.025)(1+0.025) \\
&= 1,000(1+0.025)(1+0.025)(1+0.025)(1+0.025) \\
&= 1,000(1+0.025)^4
\end{aligned}
$$

Where the number of compounding periods is 4, the principal is €1,000 and the interest rate per compounding period is $i = 0.025$.

In general, we have the following formula:

Compound Interest Formula

The amount A to which a principal P will grow after n compounding periods at a rate of i per compounding period is given by

$$A = P(1+i)^n$$

Compound Interest

Example

If €1000 is invested at 8% p.a. compounded

(i) annually,

(ii) quarterly,

(iii) monthly,

what is the amount after 5 years?

Solution

(i) $n = 5$, $i = 0.08$ per period, $P = 1{,}000$

$A = P(1+i)^n = 1{,}000(1+0.08)^5 = €\,1{,}469.33$

(ii) $n = (4)(5) = 20$, $i = \frac{0.08}{4} = 0.02$, $P = 1{,}000$

$A = 1{,}000(1+0.02)^{20} = €\,1{,}485.95$

(iii) $n = 60$, $i = \frac{0.08}{12}$ per period, $P = 1{,}000$

$A = 1{,}000\left(1+\frac{0.08}{12}\right)^{60} = €\,1{,}489.85$