MA208 Quantitative Techniques for Business

Lecture 10: Probability ctd.

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Lecture 10 - Outline

Today we will talk about

- Variance and Standard deviation of a distribution.

Variance

The variance is a measure of how spread out the distribution of a random variable is.

Variance

The variance of a random variable X, with mean $E(X)$, is

$$Var(X) = \sigma^2 = \sum_{i=1}^{k} (x_i - E(X))^2 \cdot P(x_i)$$

Example

Calculate the variance in our example from before (drawing balls from an urn containing four red balls and six black balls).

Solution

$Var(X) = \sigma^2 = \sum_{i=1}^{4} (x_i - 0.8)^2 \cdot P(x_i) = \ldots = \frac{6.4}{15} \approx 0.427$

Standard Deviation

The standard deviation, usually shown as σ, is simply the square root of variance, is another measure of the spread of the distribution.

Variance

$$\sigma = \sqrt{V(X)}$$

Example

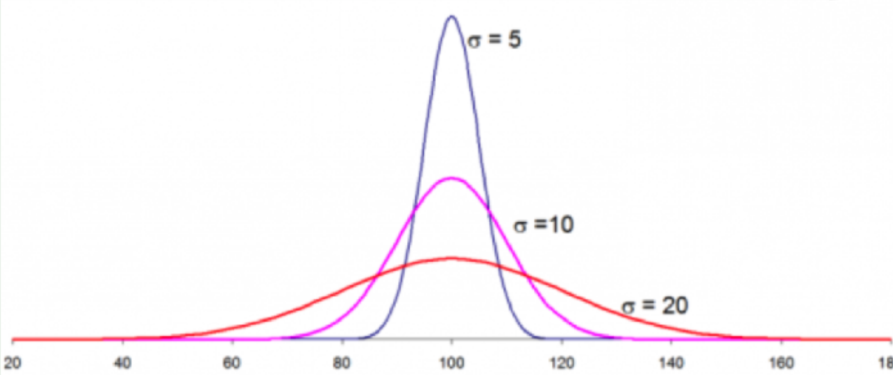
In our example above the standard deviation is

$$\sigma = \sqrt{0.427} \approx 0.653.$$

Note: The standard deviation is important in finance, where the standard deviation on the *rate of return* (the profit on an investment over a period of time) on an investment is a measure of the *volatility* (the degree of variation of a trading price series) of the investment.

Standard Deviation

Examples



Standard Deviation

Examples

Find the standard deviation σ for the following distribution.

X	8	12	16	20	24
$P(X)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

Solution

- $E(x) = \mu = 8\left(\frac{1}{8}\right) + 12\left(\frac{1}{6}\right) + 16\left(\frac{3}{8}\right) + 20\left(\frac{1}{4}\right) + 24\left(\frac{1}{12}\right) = 16$
- $V(x) = \sigma^2 = \sum_{i=1}^{5} (x_i - \mu) \cdot P(x_i)$

$$= (8-16)^2\left(\frac{1}{8}\right) + (12-16)^2\left(\frac{1}{6}\right) + (16-16)^2\left(\frac{3}{8}\right) + (20-16)^2\left(\frac{1}{4}\right) + (24-16)^2\left(\frac{1}{12}\right)$$

$$= 20$$

- $\sigma = \sqrt{20} \approx 4.472$

Standard Deviation

Knowing the mean μ and standard deviation σ of the probability distribution of X, in conjunction with Chebyshev's Rule (Lecture 4), we can make statements about the likelihood that values of X will fall within the intervals $\mu + \sigma$, $\mu + 2\sigma$ and $\mu + 3\sigma$:

$$
\begin{aligned}
P(\mu - \sigma < x < \mu + \sigma) &\geq 0 \\
P(\mu - 2\sigma < x < \mu + 2\sigma) &\geq \frac{3}{4} \\
P(\mu - 3\sigma < x < \mu + 3\sigma) &\geq \frac{8}{9}
\end{aligned}
$$

Examples

Suppose you invest a fixed sum of money in each of five Internet business ventures. Assume you know that 70 % of such ventures are successful, the outcomes of the ventures are independent of one another, and the probability distribution for the number, x, of successful ventures out of five is

x	0	1	2	3	4	5
$P(x)$	0.002	0.029	0.132	0.309	0.360	0.168

1. Find $\mu = E(x)$. Interpret the result.
2. Find $\sigma = \sqrt{Var(x)}$.
3. Graph $P(x)$. Locate μ and the interval $(\mu - 2\sigma, \mu + 2\sigma)$. Use Chebyshev's Rule to approximate the probability that x falls in this interval. Compare this result with the actual probability.

Standard Variation

Solution

(1) $\mu = E(x) = 0\,(0.002) + 1(0.29) + 2(0.132) + \dots + 5(0.168) = 3.5$

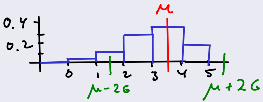
$\Rightarrow$ On average, the number of successful ventures out of 5 is 3.5.

(2) $\sigma^2 = \sum (x-\mu)\cdot p(x) = (0-3.5)^2\,(0.002) + \dots + (5-3.5)^2(.168) = 1.05$

$\sigma = \sqrt{1.05} = 1.02$

$\Rightarrow$ This value measures the spread of the probability distribution.

(3)



Chebyshev $\Rightarrow$ At least 75% of observed values fall into the interval $\mu \pm 2\sigma$.

actual probability $= p(2) + p(3) + p(4) + p(5)$

$= 0.132 + 0.309 + 0.360 + 0.168$

≈ 0.969

$\Rightarrow$ 96.9% of the probability distribution lies within 2 standard deviations of the mean.

This is consistent with Chebyshev's Rule

Example (from Exam 2016/17)

Example

A goup of students has the resources to sell any one of the following items at a market tomorrow: ice cream, hot drinks or soup. Their expected profit depends on the choice of item and the weather, and is summarized in the following table.

Weather	Ice cream	Hot drink	Soup
Sun	€ 100	€ 20	€ -30
Occasional showers	€ 30	€ 120	€ -10
Rain	€ -20	€ 70	€ 100

Assume probabilities of 30% for sun, 40% for occasional showers and 30% for rain.

(i) Which of the three items will give the maximum expected profit?

(ii) What would be the value, in €, for the students if they knew in advance how the weather is going to be?