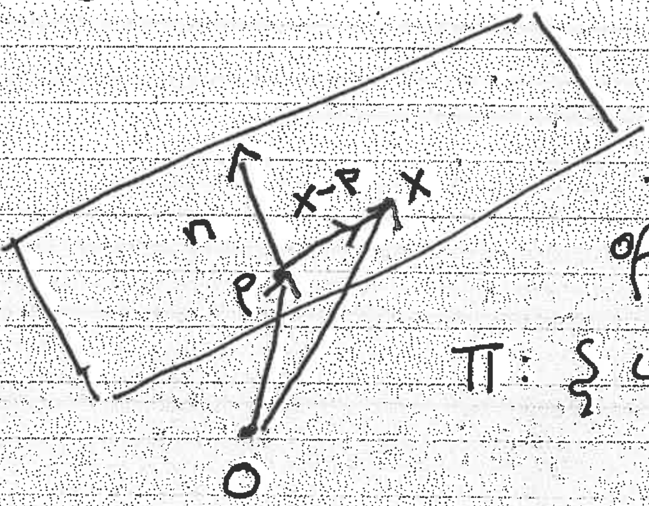
Defn: THE HYPERPLANE in $\mathbb{R}^n$ with normal $n = (n_1, \dots, n_n)$ through the ORIGIN $O = (0, 0, \dots, 0)$ ~~with normal~~ is the set of vectors (or points) $x = (x_1, x_2, \dots, x_n)$ that are perpendicular or orthogonal to n. So $n \cdot x$

$$= n_1 x_1 + n_2 x_2 + \dots + n_n x_n = 0$$

When $n = 3$ we just call it the plane ... instead of Hyperplane.

Ex. If the plane goes through some other point say $P = (p_1, p_2, p_3)$ & again has normal $n = (n_1, n_2, n_3) \neq 0$



Then the equation of the plane is

$$\Pi : \{ (x_1, x_2, x_3) = x \mid (x - P) \cdot n = 0 \}$$

So the eqn is $x \cdot n - P \cdot n = 0$

ie $x \cdot n = P \cdot n := d \in \mathbb{R}$ ie

$$n_1 x_1 + n_2 x_2 + n_3 x_3 = d$$

Ex: Find the eqn of the Plane in $\mathbb{R}^3$ with normal

$n = (2,3,1)$ Passing through the Point $P = (1,1,2)$. By above if $X = (x_1, x_2, x_3)$ lie on the Plane then

$$n \cdot X = n \cdot p \Rightarrow (2,3,1)\cdot(x_1, x_2, x_3) = (2,3,1)\cdot(1,1,2)$$

$$\Rightarrow 2x_1 + 3x_2 + x_3 = (2)(1) + (3)(1) + (1)(2) = 7$$

CHECK THAT $P = (1,1,2)$ lies on this Plane $2(1) + 3(1) + (2) = 7$ ✓

so Yes.

IT IS INTUITIVELY CLEAR THAT There is a unique Plane π through 3 points (in general position) in $\mathbb{R}^3$. How TO FIND The eqn of π.

Ex: Find The eqn of the Plane π in $\mathbb{R}^3$ Containing $(1,1,2)$, $(-3,4,6)$ & $(0,5,7)$

The eqn is of the form

$$n_1 x_1 + n_2 x_2 + n_3 x_3 = d$$

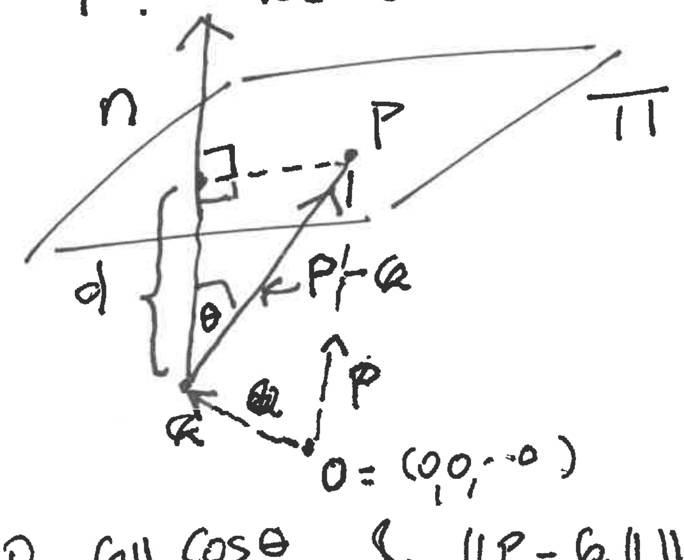
$\Rightarrow$ (i) $n_1 + n_2 + 2n_3 = d_1$ $\quad (\pi \ni (1,1,2))$

(ii) $-3n_1 + 4n_2 + 6n_3 = d_2$ $\quad (\pi \ni (-3,4,6))$

(iii) $5n_2 + 7n_3 = d_3$

So we need to solve 3 linear eqns in the 3 unknowns n_1, n_2 & n_3 (d_1, d_2 & d_3 are given Real numbers). We expect that this is just enough to yield a unique soln, e.g last year saw that generally 2 eqns in 2 unknowns have a unique soln, namely the intersection point of the 2 lines in $\mathbb{R}^2$ with the given eqns. Geometrically (generally) 3 planes in $\mathbb{R}^3$ intersect in just one pt (2 intersect in a line & this line meets the 3rd Plane in a unique pt.).

Find the distance d from a Point Q to the Plane Π with normal n & containing the Point P. We assume $Q \notin \Pi$



$d = \|P - Q\| \cos\theta$ & $\|P - Q\| \|n\| \cos\theta = (P-Q)\cdot n$

$$\therefore d = \frac{|(P-Q)\cdot n|}{\|n\|}$$