NOTE: (i) In general any 3 x 3 matrix A (or n x n) whose eigenvalues are distinct is diagonalisable

(ii) If there is a repeated eigenvalue in the 3x3 case e.g. $\lambda = 1, 1, 2$ then:

(a) if when solving $(A - 1I)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ to find the eigen vectors corresponding to $\lambda = 1$ we just get a line of solutions (ie get 2 linearly indep equs) then A is Not diagonalisable (Because missing an eigenvector to put in E)

(b) if when solving $(A - 1I)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ we get a Plane of solutions ie only one equation; then we can find 2 linearly indep eigenvectors corresponding to $\lambda = 1$ (giving us 2 columns for E) and A is diagonalisable.

(c) If all 3 eigenvalues are rep-

eg $\lambda = 1, 1, 1$ $\quad (P(\lambda) = (\lambda - 1$

Then A is not diagonalisable unless A is already diagonal

<u>More observations</u>: Recall we saw that for any upper (or low Triangular matrix $A = \begin{pmatrix} a_{11} & & * \\ & a_{22} & \\ 0 & & \ddots \ a_{nn} \end{pmatrix}$

$\det(A) = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$ ie the prod of the diagonal ~~e~~ntries of A. S in particular for a diagonal matrix

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$\det(D) = \lambda_1 \lambda_2 \lambda_3$ & so

$\det(D) = 0 \iff$ one of λ_1, λ_2 (or more) $= 0$

<u>Recall</u> <u>Also</u>: as we saw last year in the 2×2 case

$$\det(AB) = \det(A) \cdot \det(B)$$

$$\& \Rightarrow \det(A^{-1}) = \frac{1}{\det A}$$

Therefore for an $n \times n$ matrix A

which is diagonalizable $\det A$

$= \det(EDE^{-1}) = \det D = 0$

$\iff$ A has an eigen value $= 0$.

$\iff$ A^{-1} exists.

Even If A is any $n \times n$ Matrix

If $\lambda = 0$ is an eigenvalue of A

with eigen vector $v \neq 0$ (vector)

ie $Av = \lambda v = 0v = 0$

$\Rightarrow v \in \operatorname{Ker} A \Rightarrow \dim \operatorname{Ker} A \geq 1$

& Since $n = \dim \operatorname{Ker} A + \operatorname{rank} A$

$\Rightarrow \operatorname{rank} A < n$

$\Rightarrow$ A has no inverse

The converse is also true.