Compute the det A, where

$$A = \begin{pmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{pmatrix}.$$

Use the cofactor expansion down the first Column

$$\Rightarrow \det A = 3 \cdot \begin{vmatrix} 2 & -5 & 7 & 3 \\ 0 & 1 & 5 & 0 \\ 0 & 2 & 4 & -1 \\ 0 & 0 & -2 & 0 \end{vmatrix} + 0 \cdot C_{21} + 0 \cdot C_{31} + 0 \cdot C_{41} + 0 \cdot C_{51}$$

$$= 3 \cdot 2 \begin{vmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{vmatrix} = 3 \cdot 2 (-2) = -12$$

§ Row operations & Determinants:

Thm: Let A Be a square Matrix

(a) If a Multiple of one Row of A is added to another Row of A to Produce a matrix B then:

$$\det B = \det A$$

(b) If 2 rows of A are interchanged to Produce a Matrix B then

$$\det B = -\det A$$

(c) If a Row of A is Multiplied By $k \in \mathbb{R}$ $(k \neq 0)$ to Produce a Matrix

B then:

$$\det B = k \cdot \det A.$$

Ex

$$A = \begin{pmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{pmatrix}$$

Then $\begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -4 & 2 \\ 0 & 0 & -5 \\ 0 & 3 & 2 \end{vmatrix}$

$(R_2 \longleftrightarrow R_3)$ $= -\begin{vmatrix} 1 & -4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & -5 \end{vmatrix} = -(1)(3)(-5)$

$= 15.$

Ex $\begin{vmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{vmatrix} = 2 \begin{vmatrix} 1 & -4 & 3 & 4 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{vmatrix}$

$= 2 \begin{vmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & -12 & 10 & 10 \\ 0 & 0 & -3 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & -3 & 2 \end{vmatrix}$

$= 2 \begin{vmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 2(1)(3)(-6)(1) =$

$-36.$

§ Geometric explanation for the Behaviour of the determinant w.r.t Elementary Row operations.

Recall last year we saw in dim 2 (& 3?) that $\det A = \pm$ area of the parallelogram with sides given By the rows of A (& in 3 dimensions $\det A = \pm \mathrm{Vol}$ of the Box with sides given By the rows of A)

This Remains true in dimension n

So e.g. If we multiply a row of A By $k \in \mathbb{R}$ we scale the volume of the Box with sides the Rows of A By a factor of k. This explains (c) in the thm above.

Also If a Matrix A $(n \times n)$ has 2 rows the same in dimension n the Box has 2 sides collapsed onto one so it now an $n-1$ dimensional Box & thus has n-volume $= 0$

ie $\det A = 0$.

The Placing of Boxes Beside each other,

Explains the following i.e (a) in Thm above.

$$\det B = \det \begin{pmatrix} R_1 \\ \vdots \\ R_i + kR_j \\ \vdots \\ R_j \\ \vdots \\ R_n \end{pmatrix} = \det \begin{pmatrix} R_1 \\ \vdots \\ R_i \\ \vdots \\ R_j \\ \vdots \\ R_n \end{pmatrix}$$

$$+ k\det \begin{pmatrix} R_1 \\ \vdots \\ R_j \\ \vdots \\ R_j \\ \vdots \\ R_n \end{pmatrix}$$ 2 Rows the Same.

$$= \det A + k(0) = \det A$$

§ Finally: Similar to last year where $n = 3$

We have for any $n \times n$ Matrix A

with $\det A = |A| \neq 0$

$$A^{-1} = \frac{1}{|A|}\left(C_{ij} \right)^t$$

where for a Matrix B, its transpose B^t

is obtained from B by writing the Rows

of B as the Columns of B^t.