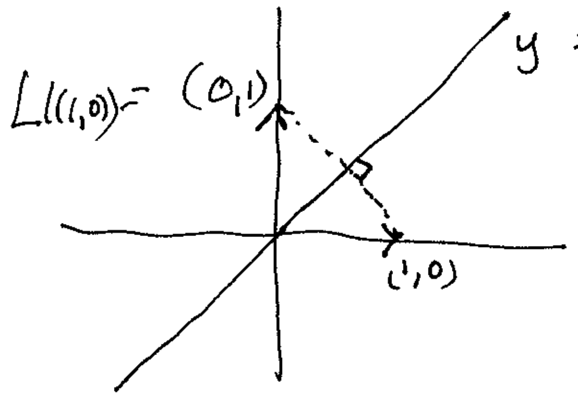
Non Square matrices:

Recall: The LAST DAY we saw how to obtain the Matrix for a Linear Transformation. We only need to find $L((1,0))$ & $L((0,1))$. If $L((1,0)) = (a,c)$ & $L((0,1)) = (b,d)$

Then $L \underset{1:1}{\longleftrightarrow} A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Ex: Let L be the linear transformation of Reflection in the line $y = x$



Then $L((1,0)) = (0,1)$ $(=(a,c))$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

& $L((0,1)) = (1,0)$ $(= b,d)$

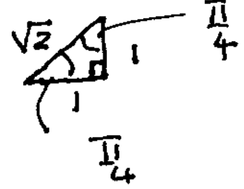
Now to find e.g. $L((2,3))$ place $(2,3)$ Behind A as a Column & Muliply Both rows of A By the Column $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ie

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$ ie $L((2,3)) = (3,2)$

Similarly we saw that the matrix for a Rotation (about the origin) $\circlearrowleft$ By θ (denoted By R_θ)

is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. So Find the

image of the point or vector $(3,5)$ when rotatated about $(0,0)$ By $\frac{\pi}{4}$.

Note $\cos \pi/4 = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ (

)

$$\therefore R_{\frac{\pi}{4}}((3,5)) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -2/\sqrt{2} \\ \frac{8}{\sqrt{2}} \end{pmatrix} \Leftrightarrow (-\sqrt{2}, 4\sqrt{2})$$

Recall Also: We saw that Matrix multiplication corresponds to composition of linear Tranformations. For example

Let $f((x,y)) = (\cancel{x}+2y, 3y-x)$

(In class I didn't spot the $3y$ first & $-x$ second

Correction: $\longrightarrow \; = (x+2y, -x+3y)$

$$\longleftrightarrow \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

8 Let $g((x,y)) = (x-2y,\ x+y)$

ie $\longleftrightarrow \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$

Then the Matrix for the composition of g after (or with) f denoted By

$(g \circ f)((x,y)) := g((f((x,y)))$

Read as g of f

ie $(x\ y) \xrightarrow[f]{\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}} f((x,y)) \xrightarrow[g]{\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}} g(f(x,y))$

is $\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

e.g. $(g \circ f)((1,1)) = \underbrace{\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 & -4 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 5 \end{pmatrix} \longleftrightarrow (-5, 5)$

In General we define an $m \times n$ matrix to Be an array of (mn) numbers arranged in m rows and n Columns.

eg. $\underset{2\times 3}{A} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix}$ $\quad \underset{3\times 1}{B} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$

$\underset{2\times 2}{C} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ $\quad \underset{1\times 3}{D} = \begin{pmatrix} 4 & -1 & 2 \end{pmatrix}$

$\underset{2\times 3}{E} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \end{pmatrix}$ $\quad F = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}$

ADDITION OF NON SQUARE MATRICES:

Can only ADD IF SAME SHAPE ie $m\times n + m\times n$ ✓
& just ADD corresponding entries

eg $A + E = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 3 & 2 \\ -1 & 6 & 2 \end{pmatrix} = E + A.$

We can extend the definition of multiplying a Row of a 2×2 By a Column of a 2×2 to the case where a Row has the same number of elements as the Column.

e.g. $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = (1)(4) + (2)(5) + (3)(6) = 32$

So can multiply an $\underset{m \times n}{A}$ by an $\underset{n \times p}{B}$ to get an $m \times p$ matrix AB

Because the no of elements in a Row of $A = n$ & n = the number of elements in a column of B.

Rule:

$$\underset{m \times \underbrace{\cancel{n}\ \cancel{n}}_{\text{equal}} \times p}{A\ B} = \underset{m \times p}{AB}$$

$\doteq$ multiply all rows of A By all Cols of A & Place answer in corresponding position.

Ex: Can we multiply A By C ?

A is 2×3 & C is 2×2

$$2 \times \underbrace{3 \quad 2}_{\text{not equal}} \times 2$$

So No!

$$\underset{2 \times \underbrace{3\ 3}_{\text{equal}} \times 1}{A\ B}$$

Yes & answer is 2×1

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} (1)(-1) + (2)(3) + (1)(0) \\ (0)(-1) + (4)(3) + (1)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

$$\underset{2 \times \underbrace{2\ 2}_{\text{equal}} \times 3}{C\ A}$$

Yes & answer is 2×3

$$\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 14 & 5 \end{pmatrix}$$

ED No!

$2\times\underbrace{3\quad 1}_{\text{not equal}}\times 3$

BE No! But EB Yes $= 2\times 1$

exercise

$3\times\underbrace{1\quad 2}_{\text{Not equal}}\times 3$

$2\times\underbrace{\cancel{3}\quad \cancel{3}}_{\text{equal}}\times 1$

BD Yes & answer $= 3\times 3$

$3\times\underbrace{1\quad 1}_{\text{equal}}\times 3$

$$= \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}\begin{pmatrix} 4 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 1 & -2 \\ 12 & -3 & 6 \\ 0 & 0 & 0 \end{pmatrix}$$