Recall: A Linear map (or transformation) of the plane $\mathbb{R}^2$ is a function

$$L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$: v \longrightarrow L(v) \qquad \text{s.t.}$$

(i) $L(u+v) = L(u) + L(v) \quad \forall\, u, v \in \mathbb{R}^2$

& (ii) $L(ru) = r\,L(u) \quad \forall\, u \in \mathbb{R}^2,\ r \in \mathbb{R}$

so L is given by the rule:

$$L((x,y)) = x\,L((1,0)) + y\,L((0,1))$$
$$= x\,(a,c) + y\,(b,d)$$
$$= (ax + by,\ cx + dy)$$

(By writing $(x,y) = x(1,0) + y(0,1)$ & using (i) & (ii))

so $L \longleftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$a, b, c, d \in \mathbb{R}$ determined by $L((1,0)) := (a,c)$
$L((0,1)) := (b,d)$

Note: $L((0,0)) = (a(0) + b(0),\ c(0) + d(0)) = (0,0)$ always.

Ex: $L((x,y)) = (2x+3y,\ x-y)$ is Linear

$(a = 2,\ b = 3,\ c = 1,\ d = -1)$

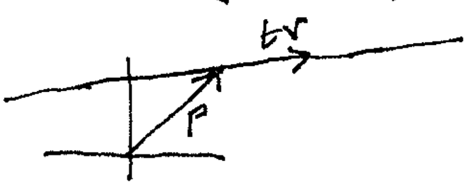
$L((x,y)) = (2\underline{\underline{\underline{xy}}},\ x+y)$ is Not

$L((x,y)) = (7x+3y,\ 4x+\underline{\underline{\underline{5}}})$ is Not

$(\ L((0,0)) = (0,5) \neq (0,0)\ )$

We also saw that $L(P + tv) = L(P) + t\,L(v)$

$\Rightarrow$ L sends the line through P in direction v



to " " " $L(P)$ " " $L(v)$

(for $L(v) \neq (0,0)$)

To recap on some of the material covered so far we do Questions 6 & 7 from a sample online homework.

$\boxed{Q.6}$ Is the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$f((x,y)) = (2x + 2y, \; y - x)$ linear?

Recall we saw that if f is linear then

$$f((x,y)) = (ax + by, \; cx + dy)$$ where

$a, b, c, d \in \mathbb{R}$ defined by $f((1,0)) = (a,c)$
& $f((0,1)) = (b,d)$

So the above f is of this form with

$a = 2, \quad b = 2, \quad c = -1 \quad \& \quad d = 1$

(Note! $y - x = -x + y$). & is therefore Linear.

$\boxed{Q7}$ Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be linear with

$f((1,0)) = (3,4) = (a,c) \implies a = 3, \; c = 4$

& $f((0,1)) = (1,1) = (b,d) \implies b = 1, \; d = 1$

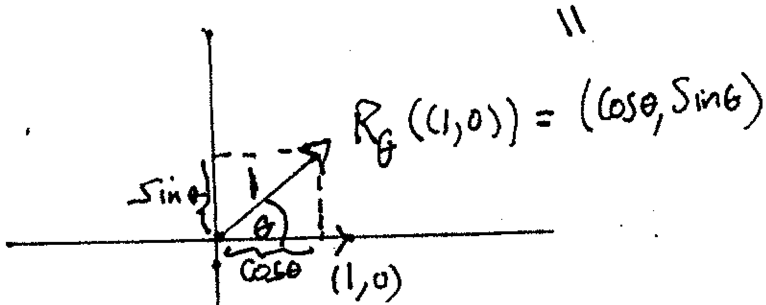
$\therefore f((x,y)) = (ax + by, \; cx + dy) = (3x + y, \; 4x + y)$

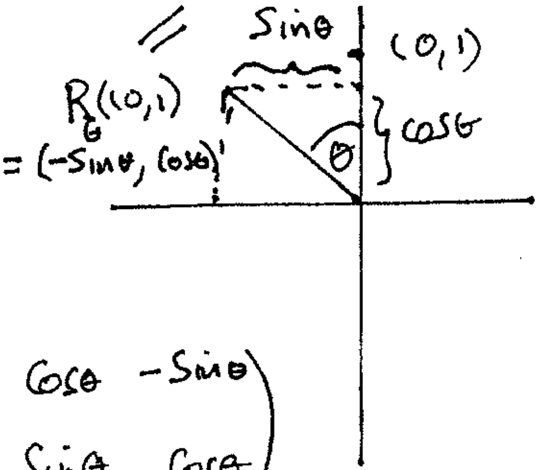
$\implies f((2,3)) = (3(2) + 3, \; 4(2) + 3) = (9, 11)$

Other important linear maps are Rotations (R_θ) anti-clockwise about $(0,0)$ By an angle θ (sends lines through $(0,0)$ to " " ")

Lets Find its MATRIX ie need to find

$$R_\theta((1,0)) := (a,c) \quad \& \quad R_\theta((0,1)) := (b,d)$$

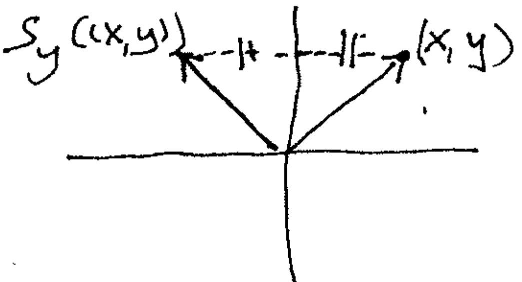




$\therefore$ R_θ has matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

<u>Exercise</u>: Find the matrix for Reflection of the Plane in the y-axis (denoted S_y)

(ie Find $S_y((1,0)) = (a,c)$
$S_y((0,1)) = (b,d)$)



MATRIX Algebra encodes in Algebra The geometry of Linear Transformations so e.g. can Be coded on a computer e.g PIXAR uses matrices for Rotating & Projecting images.

1. Addition of Matrices (corresponds to addition of 2 linear transformations again linear so has a matrix)

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$,

($A = (a_{ij})$, $B = (b_{ij})$ where a_{ij} is the entry or number in the i-th row & j-th column of A.)
so the first subscript (i) tells you what row you're in
& 2nd " (j) " " " column " "

Defn: $A + B := \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$

e.g. $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 3.5 & 2 \\ 0 & 1 \end{pmatrix}$

$A+B = \begin{pmatrix} 4.5 & 4 \\ 3 & -3 \end{pmatrix} = B+A$ (Addition is commutative)

& O (the zero matrix) is a special matrix

$O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ then $A + O = O + A = A$
for all matrices A

2. Multiplying a matrix by a number $r \in \mathbb{R}$ ($\leftrightarrow$ Scaling a linear transformation)

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $rA = \begin{pmatrix} ra & rb \\ rc & rd \end{pmatrix}$ e.g. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; $2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$
$r = 2$