ALGEBRA

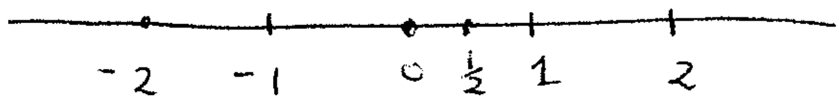
§ SECTION 1: VECTORS, MATRICES & LINEAR ALGEBRA

TEXT: By M. LAWSON ON BLACKBOARD (FREE!)

Recall

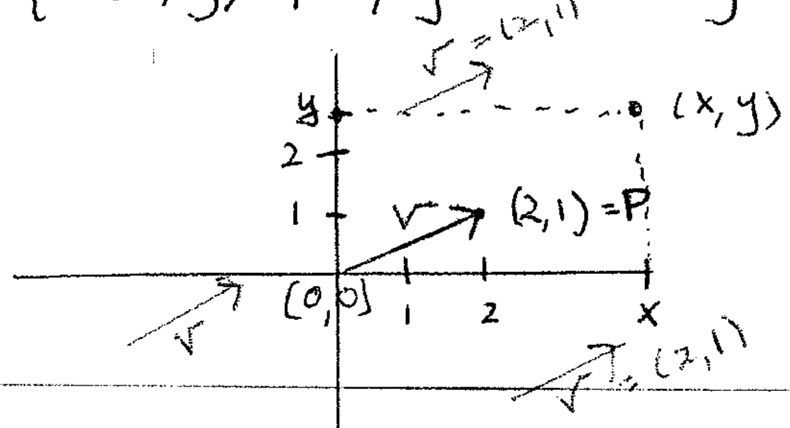
$\mathbb{R} = \{ \text{all Real numbers} \}$

$\longleftrightarrow$ PICTURE



$\mathbb{R}^2 := \{ (x, y) \mid x, y \in \mathbb{R} \}$

$\longleftrightarrow$ PICTURE



Any element e.g. $(2, 1) \in \mathbb{R}^2$ can Be Pictured & though of as :

(i) The Point $P = (2,1)$ (OR Q, R, S Uppercase Letters for Points)

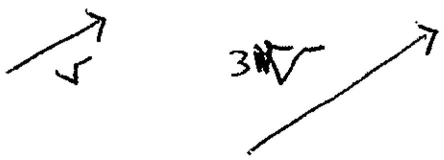
OR (ii) The Vector $v = (2,1)$ With Base $(0,0)$ & ending (OR tip) at $(2,1)$ (OR u, w lower case letters for vectors)

So v has a direction & a magnitude (OR Length) denoted $\|v\|$. If $v = (x, y)$ then By Pythag. obviously $\|v\| = \sqrt{x^2+y^2}$.

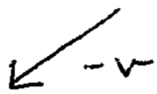
Since a vector v is characterised By its length & direction we can Base v at any point or position in The Plane $\mathbb{R}^2$ when we picture it. NOT NECC. TO Base it at $(0,0)$ the origin.

OPERATIONS on VECTORS:

(i) Scalar Multiplication: Let $v = (x,y)$ & $r \in \mathbb{R}$ then we can Scale v By r ie $rv := (rx, ry)$ is r times as long with same direction as v eg. $3(2,1) = (6,3)$



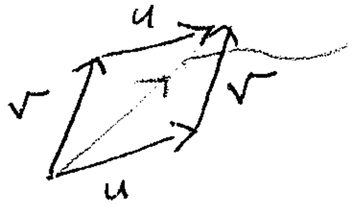
When $r = -1$ we write $-1v = -v$



(opposite direction)

(ii) Addition Let $u = (x, y)$ & $v = (x', y')$

then $u + v := (x + x', y + y') = v + u$



ie Place the Base of v at tip of u & go from $(0,0)$ to the tip of u. (or vice-versa) eg. $u = (2, 3)$ $v = (2, 1)$

$u + v = (4, 4)$

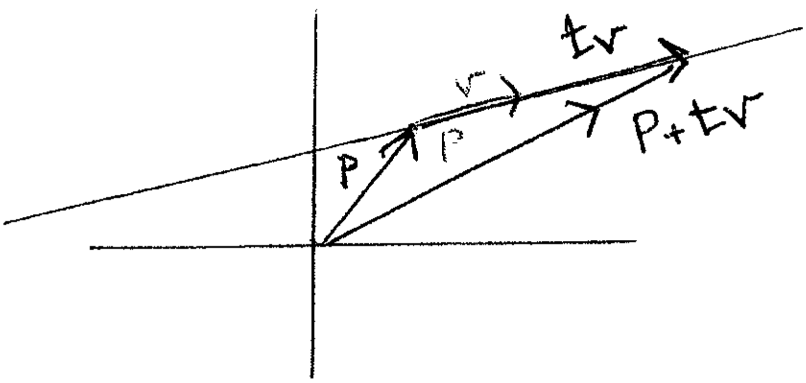
SPECIAL VECTORS: THE ZERO VECTOR $0 := (0,0)$

$e_1 := (1, 0)$ $e_2 = (0, 1)$ so any vector $v = (x, y)$ can Be expressed as $v = xe_1 + ye_2$

§ THE Parametric equation of a line $\ell \subseteq \mathbb{R}^2$

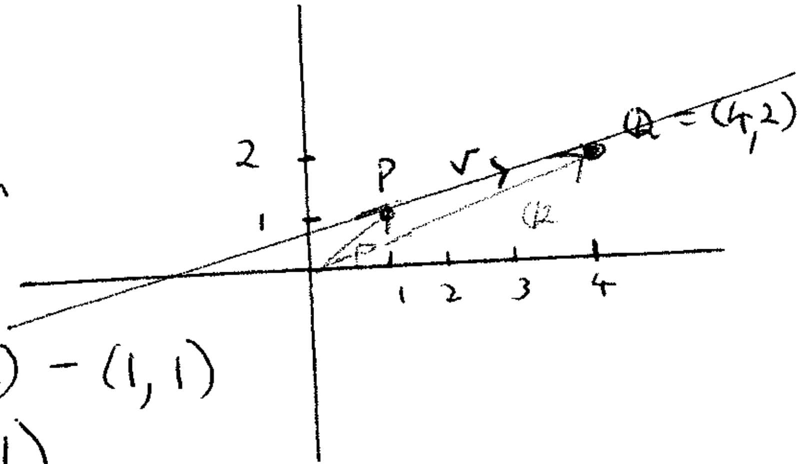
Let ℓ Be the line in the direction v containing the point P. The

$$\ell := \{ P + tv \mid t \in \mathbb{R} \}$$ (the Parameter)



as t Ranges over $\mathbb{R}$ all real no.s we draw out the line

Ex: Find the Parametric equation of the line containing the points $P = (1,1)$ & $Q = (4,2)$.



Note $P + v = Q$

So v the direction of ℓ

So $v = Q - P = (4,2) - (1,1)$
$= (3,1)$

$\therefore \ell = \{ P + tv \mid t \in \mathbb{R} \}$

$= \{ (1,1) + t(3,1) \mid t \in \mathbb{R} \}$

$= \{ (1+3t, 1+t) \mid t \in \mathbb{R} \}$

So for the general point (or vector) (x,y) on ℓ we have $x = 1+3t$ $y = 1+t$

$\Rightarrow t = \frac{x-1}{3} = \frac{y-1}{1}$ is the Parametric eqn.

Note $\Rightarrow 1(x-1) = 3(y-1)$

$\Rightarrow x - 3y + 2 = 0$

AT SCHOOL: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$= y - 1 = \frac{2-1}{4-1}(x-1) \Rightarrow x - 3y + 2 = 0$ ✓