

A sequence of simplicial complexes

$$K_{\Sigma_1} \subseteq K_{\Sigma_2} \subseteq K_{\Sigma_3} \subseteq \dots \subseteq K_{\Sigma_t}$$

gives rise to a sequence of linear homomorphisms of vector spaces, for each degree $n \geq 0$,

$$H_n(K_{\Sigma_1}) \longrightarrow H_n(K_{\Sigma_2}) \longrightarrow H_n(K_{\Sigma_3}) \longrightarrow \dots \longrightarrow H_n(K_{\Sigma_t})$$

Defn The degree n persistent

Betti number β_n^{ij} is defined

to be the rank of the composite linear homomorphism

$$H_n(K_{\Sigma_i}) \longrightarrow H_n(K_{\Sigma_j})$$

for $i \leq j$. If $i > j$ define $\beta_n^{ij} = 0$.

$$\beta_1^{**} = \begin{pmatrix} 2 & 2 & 2 & 1 & 0 \\ 0 & 3 & 3 & 2 & -1 \\ 0 & 0 & 5 & 2 & -1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$



β_1^{**} describes 1-dimensional holes