<u>Aim</u> State Leray's nerve theorem as motivation for studying the simplicial complex K_Σ.

Let $\mathcal{U} = \{U_1, U_2, \cdots, U_n\}$ be a collection of sets. The <u>nerve</u> $N\mathcal{U}$ is a simplicial complex with

$$V = \{U_1, U_2, \cdots, U_n\}$$

and with a d-simplex

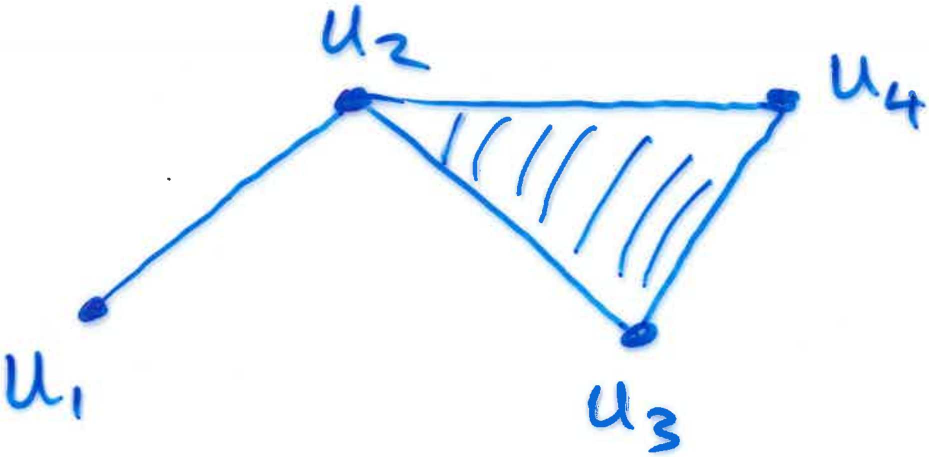
$$\sigma = \{U_{i_0}, U_{i_1}, \cdots, U_{i_d}\}$$

whenever

$$U_{i_0} \cap U_{i_1} \cap \cdots \cap U_{i_d} \neq \emptyset .$$

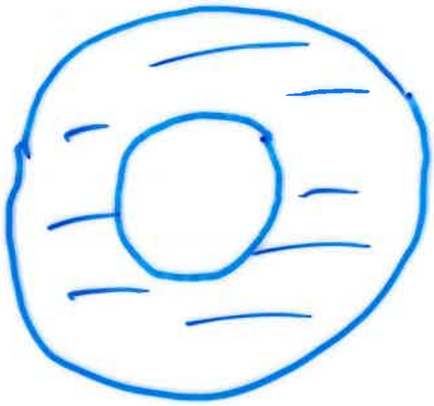
<u>Example 1</u>

$$\mathcal{U} = \{U_1 = \{1,2,3\}, U_2 = \{3,4,5\}, U_3 = \{4,5,6\}, U_4 = \{5,6,7\}\}$$



<u>Example 2</u> Consider

$$X = \{ z \in \mathbb{C} : 1 \leq |z| \leq 2 \}$$

$X =$ 

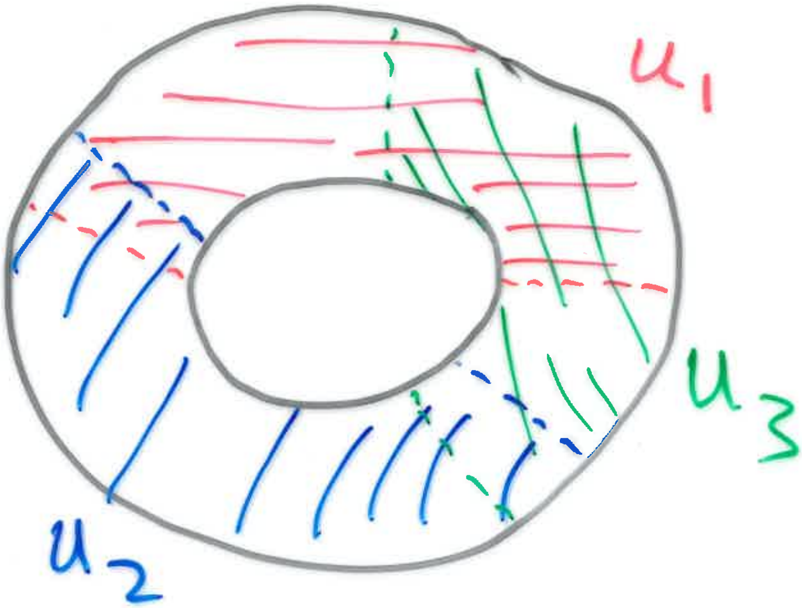
Consider

$$U_1 = \{ z \in X : 0 < \operatorname{Arg}(z) < \frac{5\pi}{6} \}$$

$$U_2 = \{ z \in X : \frac{4\pi}{6} < \operatorname{Arg}(z) < \frac{11\pi}{6} \}$$

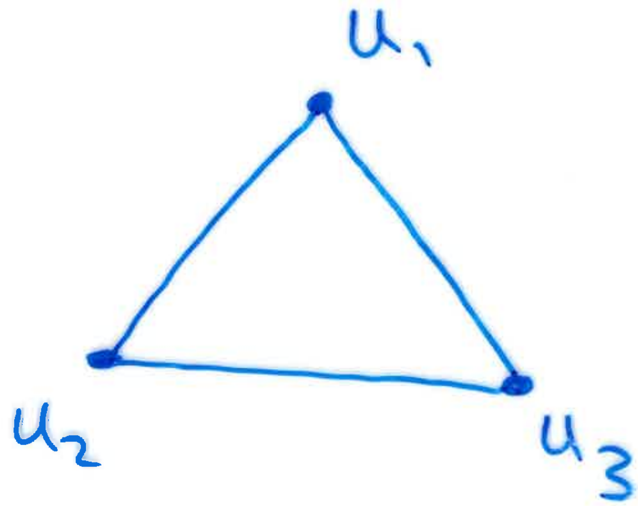
$$U_3 = \{ z \in X : -\frac{2\pi}{6} < \operatorname{Arg}(z) < \frac{3\pi}{6} \}$$

$X =$



So $\mathcal{U} = \{U_1, U_2, U_3\}$ is an <u>open cover</u> of X. i.e. each U_i is open and $X = U_1 \cup U_2 \cup U_3$

$N\mathcal{U} =$



Note that $N\mathcal{U}$ is homotopy equivalent to the circle S^1, as is X. So $N\mathcal{U}$ is homotopy equiv. to X.

Recall Two continuous functions

$$f, g : X \longrightarrow Y$$

are homotopic if there is a continuous function

$$H : X \times [0,1] \longrightarrow Y, \quad (x,t) \mapsto H(x,t)$$

with

$$\left.\begin{aligned} f(x) &= H(x,0) \\ g(x) &= H(x,1) \end{aligned}\right\} \text{ for all } x \in X.$$

Recall Two spaces X and Y are homotopy equivalent if there exist continuous functions

$$f : X \to Y, \quad g : Y \to X$$

with

$f \circ g : Y \longrightarrow Y$ homotopic to the identity $1 : Y \to Y$

$g \circ f : X \to X$ " " $1 : X \to X$

Recall The Euler characteristic $\chi(X)$ equals $\chi(Y)$ whenever X is homotopy equivalent to Y.

Defn A space X is contractible if it is homotopy equivalent to a space $\{1\}$ with just one point.

Theorem (Leray) If $\mathcal{U} = \{U_1, \dots, U_n\}$ is an open cover of a compact space X such that every non-empty intersection of finitely many sets in $\mathcal{U}$ is contractible, then X is homotopy equivalent to the Nerve $|N\mathcal{U}|$.