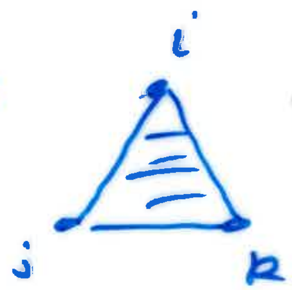
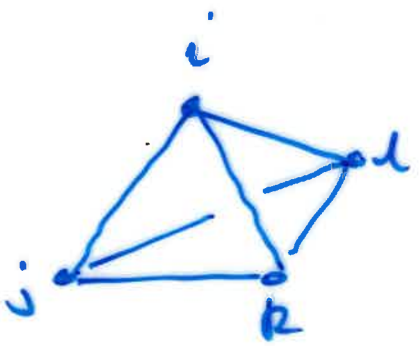
<u>Recall</u> Given an $n \times n$ symmetric matrix $D = (d_{ij})$ of distances/dissimilarities, and given $\varepsilon > 0$, we construct the simplicial K_ε with:

- vertex set $V = \{1, 2, \dots, n\}$
- an edge  whenever $d_{ij} \leq \varepsilon$
- a 2-simplex  whenever $d_{ij} \leq \varepsilon$, $d_{ik} \leq \varepsilon$, $d_{jk} \leq \varepsilon$
- a 3-simplex  whenever $d_{uv} \leq \varepsilon$ for all $u \neq v \in \{i, j, k, l\}$
- a d-simplex for each $\sigma = \{v_0, v_1, \dots, v_d\} \subseteq V$ with $d_{uv} \leq \varepsilon$ for all $u \neq v \in \sigma$.

Remark An alternative view is that K_Σ is a graph on n vertices with a d-simplex for each complete subgraph on $d+1$ vertices.

Example 200 people were asked to visit Galway harbour at their convenience once during a given 2-week period. They were asked to record the height of the water on their arrival, and then again 2 hours later, and again 4 hours after their arrival. Each person returns their recordings $(h_0, h_2, h_4) \in \mathbb{R}^3$. From the set $S = \{ x_i = (h_{i0}, h_{i2}, h_{i4}) : 1 \le i \le 200 \}$

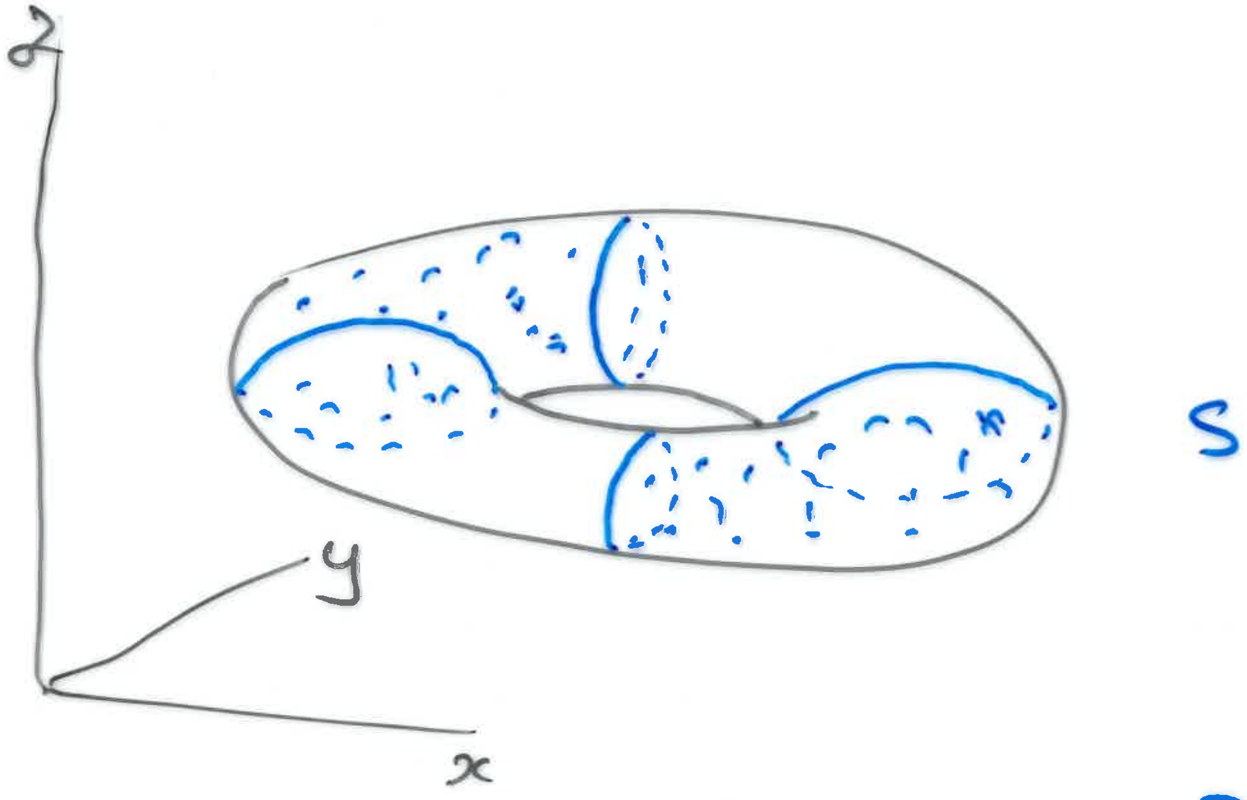
a data analyst might construct

$D = (d_{ij})$ with

$$d_{ij} = \| x_i - x_j \| \quad \text{(Euclidean metric)}$$

and then view the graph of the simplicial complex K_ε for various $\varepsilon \geq 0$.

<u>Example</u> Consider a sample S of 750 points selected at random from two "quarter segments" of a torus.



$S \subset \text{Torus} \subset \mathbb{R}^3$

Note that any linear tranformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ will loose significant geometric information about S since a torus can't be embedded in the plane.

Note :

$$\text{Torus} = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} x = \cos(s)(1+\cos t) \\ y = \sin(s)(1+\cos t) \\ z = \sin(t) \\ 0 \leq s, t \leq 2\pi \end{array} \right\}$$

From S let's compute a distance matrix $D = (d_{ij})$, Euclidean metric, and view the graph of K_ε for various $\varepsilon \geq 0$.