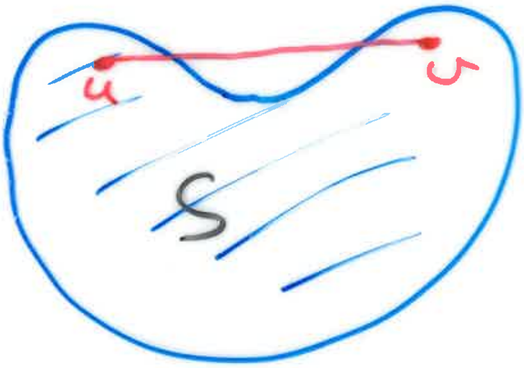
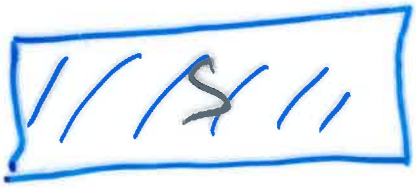
Defn A set $X \subseteq \mathbb{R}^n$ is said to be convex if, for any $u, v \in S$, the straight line from u to v lies entirely in S.

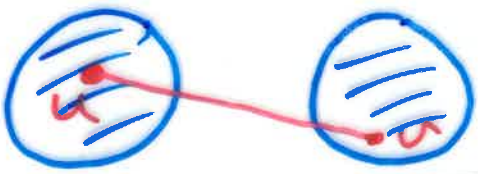
Example



$\subseteq \mathbb{R}^2$ Not Convex



$\subseteq \mathbb{R}^2$ Convex

$S =$ 

$\subseteq \mathbb{R}^2$ Not Convex

$\subseteq \mathbb{R}^2$ Convex

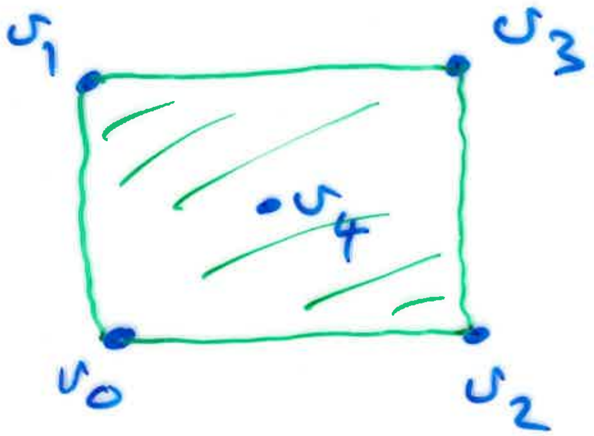
Suppose $v_0, v_1, \dots, v_k \in \mathbb{E}^n$

Let

$$C = \mathrm{Conv}(v_0, v_1, \dots, v_k)$$

denote the "smallest" convex set in $\mathbb{E}^n$ containing $v_0, v_1, \dots, v_k$.

<u>Example</u> $v_0 = (0,0)$, $v_1 = (0,1)$, $v_2 = (1,0)$, $v_3 = (1,1)$, $v_4 = (\frac{1}{2}, \frac{1}{2}) \in \mathbb{R}^2$

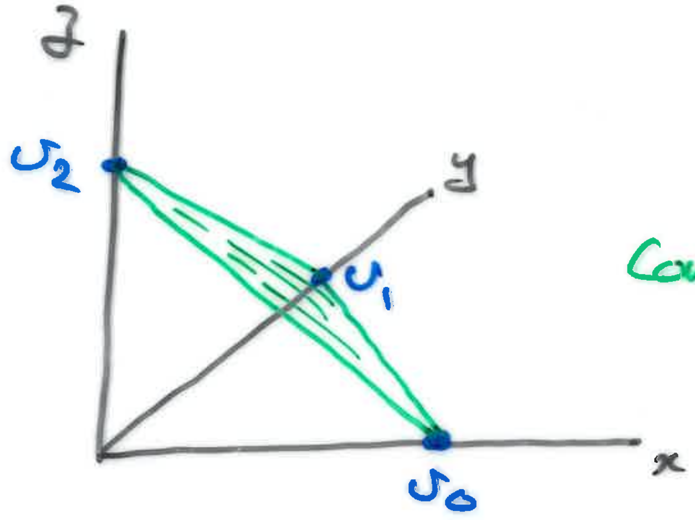


$= \mathrm{Conv}(v_0, v_1, v_2, v_3, v_4)$

Example $v_0 = (1,0,0)$

$v_1 = (0,1,0) \quad \in \mathbb{R}^3$

$v_2 = (0,0,1)$



$\mathrm{Conv}(v_0, v_1, v_2)$

Defn We call $C = \mathrm{Conv}(v_0, v_1, \dots, v_k)$ the convex hull of the points $v_0, v_1, \dots, v_k \in \mathbb{R}^n$.

In general

$$C = \mathrm{Conv}(v_0, v_1, \dots, v_k)$$

consists of all those points in $\mathbb{R}^n$ of the form

$$\lambda_0 v_0 + \lambda_1 v_1 + \cdots + \lambda_k v_k$$

with $\lambda_i \in \mathbb{R}$, $\lambda_i \geq 0$ and

$$\lambda_0 + \lambda_1 + \cdots + \lambda_k = 1.$$

Defn Let $v_0, v_1, \ldots, v_k \in \mathbb{E}^n$ be linearly independent. We call

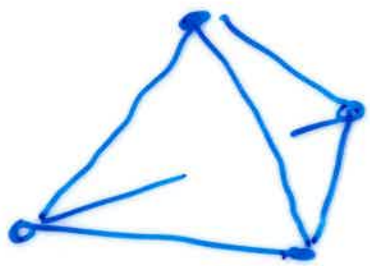
$$C = \mathrm{Conv}(v_0, v_1, \ldots, v_k)$$

a simplex of dimension k, or k-simplex.

0-simplex = point 

1-simplex = line segment 

2-simplex = solid triangle 

3-simplex = solid tetrahedron 

Simplexes have "faces".

If A and B are simplexes, and if the vertices of A form a subset of the vertices of B then we say that A is a face of B.

Example A 3-simplex has

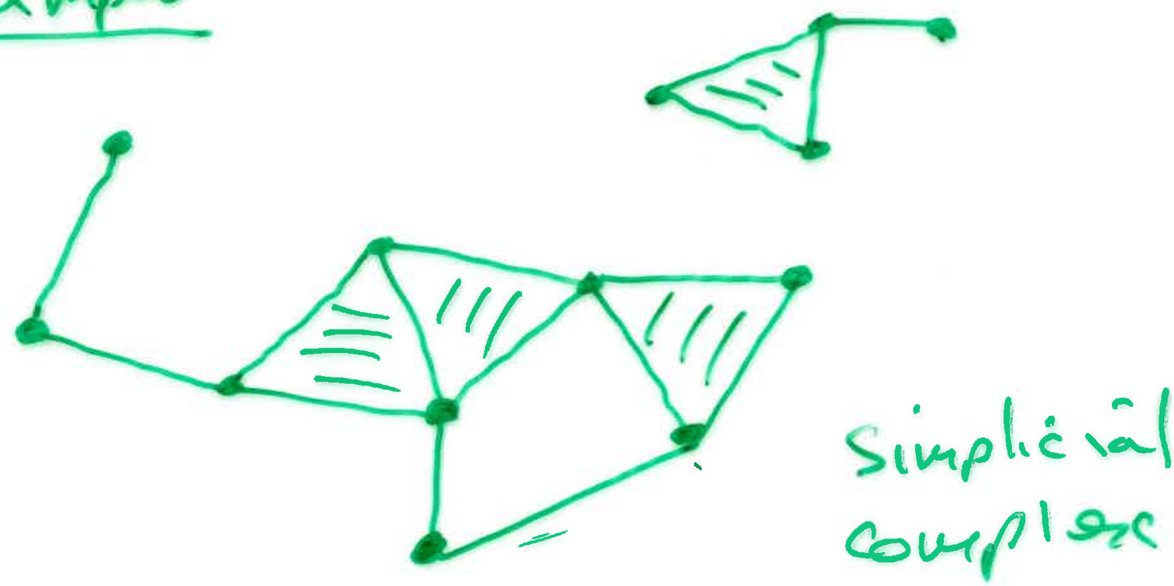
- four faces of dimension 2
- six faces of dimension 1
- four faces of dimension 0
- one face of dimension 3

Defn A finite collection of simplexes in $\mathbb{E}^n$ is called a <u>simplicial complex</u> if:

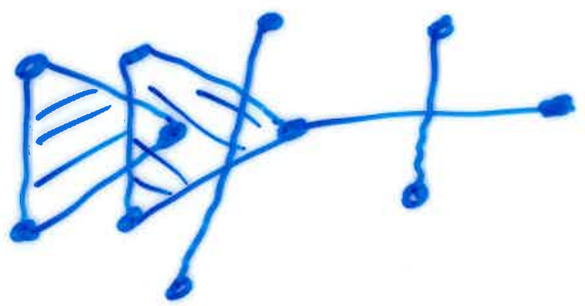
i) whenever a simplex lies in the collection, then so too do all of its faces;

ii) whenever two simplexes of the collection intersect, they do so in a common face.

Example



Non-example



A simplicial complex is a subset of $\mathbb{E}^n$ and is thus a subspace of $\mathbb{E}^n$ with the subspace topology.

We let $K, L, \ldots$ denote simplicial complexes.

We write $|K|, |L| \ldots$ to denote the corresponding subspace of $\mathbb{E}^n$.

$$\boxed{\begin{array}{l}\underline{\text{Defn}}\ \text{Let } X \text{ be a topological space. A } \underline{\text{triangulation}} \text{ of } X \\ \text{consists of a simplicial complex } K \text{ and a homeomorphism} \\ \qquad h : |K| \longrightarrow X.\end{array}}$$