<u>Aim</u> Show that Peano's function

$$f : [0,1] \longrightarrow \Delta$$

is surjective.

$$f = \lim_{n\to\infty} f_n$$

It should be clear that Δ equals the union of Image(f) with the accumulation points of Image(f).

So we just need to show that Image(f) contains all its accumulation points.

i.e. we need to show that

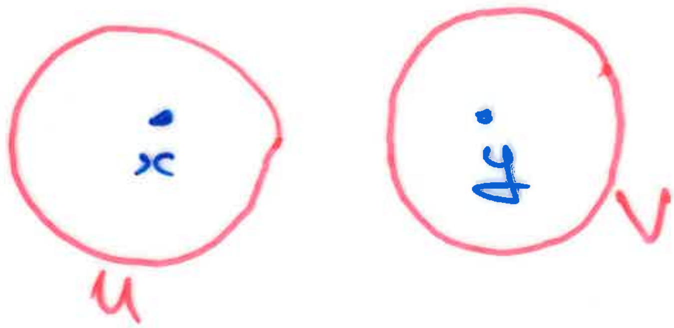
Image(f) is closed.

We know that $[0,1]$ is compact, and hence (by one of our propositions) that $f([0,1])$ is compact.

$$\underset{\text{Image}(f)}{\overset{\|}{f([0,1])}}$$

We just need to show that "compact" implies "closed" under a suitable hypothesis.

<u>Defn</u> A topological space X is said to be Hausdorff if for any distinct $x, y \in X$ there exists open sets U, V in X with $x \in U$, $y \in V$ and $U \cap V = \emptyset$.



<u>Example</u> $\mathbb{R}$ with standard topology is Hausdorff. So too is $\mathbb{R}^n$.

<u>Example</u> $\mathbb{Z}$ with the cofinite topology (i.e. $U \subseteq \mathbb{Z}$ is open if $\mathbb{Z} \setminus U$ is finite or $U = \emptyset$) is not Hausdorff.

<u>Exercise:</u> If X is Hausdorff and if Y is homeomorphic to X then Y is Hausdorff.

<u>Proposition</u> A compact subset of a Hausdorff topological space is closed.

<u>Proof</u> Let X be a Hausdorff topological space. Let A be a compact subset of X.

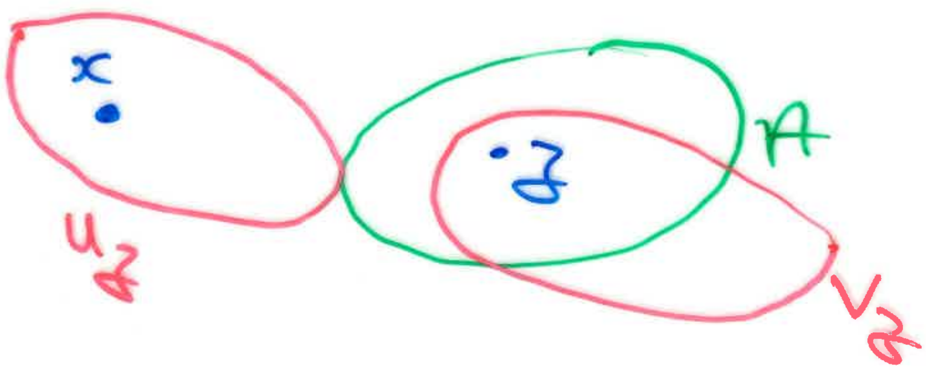
Just need to show that A contains all its accumulation points.

Let

$$x \in X \setminus A .$$

Just need to show that x is not an accumulation point of A.

Let $z \in A$. Since X is Hausdorff we can find open sets U_z, V_z such that $x \in U_z$, $z \in V_z$ and $U_z \cap V_z = \phi$.



we have a collection of open sets

$$\{ V_\zeta \}_{\zeta \in A}$$

whose union contains A. But A is compact, so (!) we can find a finite collection of points

$$\zeta_1, \zeta_2, \ldots, \zeta_k \in A$$

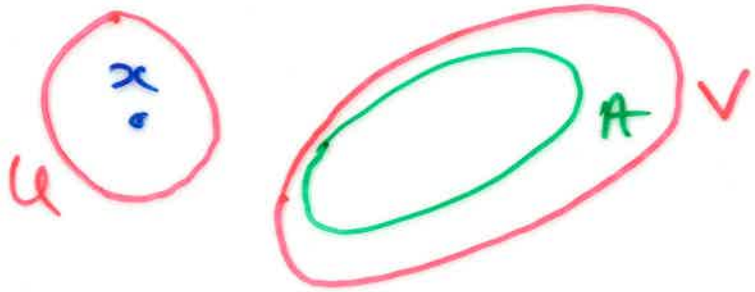
with

$$A \subseteq V_{\zeta_1} \cup V_{\zeta_2} \cup \cdots \cup V_k =: V$$

Now V is disjoint from the following finite intersection

$$U := U_{\zeta_1} \cap U_{\zeta_2} \cap \cdots \cap U_{\zeta_k}.$$

But U is open since it is a finite intersection of open sets.



Since $U \cap V = \emptyset$ we see that x is <u>not</u> an accumulation point of A. Hence A contains all its accumulation points.

Hence A is closed.

Next aims:

- Give a precise definition of the Euler characteristic of a topological space.
- Give a flavour of the ingredients of the proof that the Euler characteristic is a topological property.
- Give an application of the Euler characteristic in Economics.