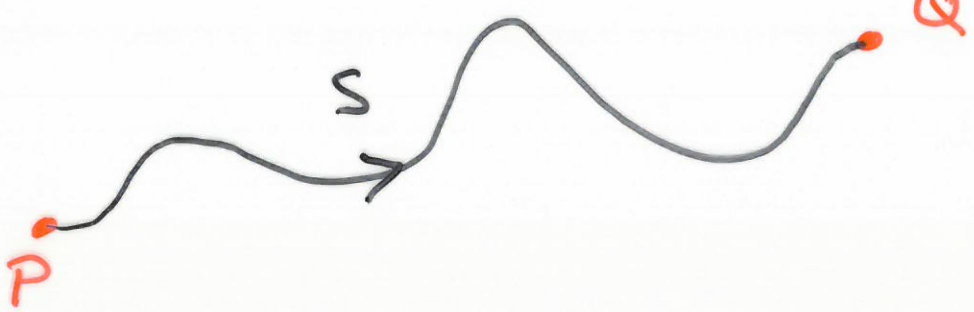
MA2286 Test: Wed 10 October

All material up to and including Section 7. Five questions, of equal weight. 50 minutes.

The Fundamental Theorem of Calculus

Let ω be a differential 0-form on n-dimensional space. Let S be a curve in $\mathbb{R}^n$ from P to Q.



Theorem

$$\int_S d\omega = \int_{\partial S} \omega$$

Example Evaluate

$$I = \int_S (y^3 + 2x)\,dx + 3xy^2\,dy$$

where S is the straight line from $P = (0,0)$ to $Q = (1,2)$.

Sol^n (using FTC)

Consider

$$\omega = xy^3 + x^2$$

Then

$$d\omega = (y^3 + 2x)\,dx + 3yx^2\,dy$$

So

$$I = \int_S d\omega = \int_{\partial S} \omega \overset{\text{by}}{\underset{\text{definition}}{=}} \omega(Q) - \omega(P)$$

$$= 9 - 0 = 9.$$

Alternative Solution

The points $(x=t, y=2t)$ trace out the straight line segment from $P=(0,0)$ to $Q=(1,2)$ as t goes from 0 to 1.

$x=t$ $\qquad\qquad$ $y=2t$

$dx=dt$ $\qquad\qquad$ $dy=2dt$

$$I=\int_0^1 \left((2t)^3+2t\right)dt + 3t(2t)^2\,2dt$$

$$=\int_0^1 (32t^3+2t)\,dt$$

$$=\frac{32t^4}{4}+\frac{2t^2}{2}\Big|_0^1$$

$$=9.$$

<u>Problem</u> Evaluate

$$I = \int_S (6xy^2 - y^3)\,dx + (6x^2y - 3xy^2)\,dy$$

where S is <u>some</u> curve

from $P = (1,1)$ to $Q = (3,4)$.

<u>Sol^n</u> Try to find $\omega = F(x,y)$

such that

$$d\omega = F_x\,dx + F_y\,dy$$

$$= (6xy^2 - y^3)\,dx + (6x^2y - 3xy^2)\,dy$$

well

$$F(x,y) = 3x^2y^2 - xy^3 + g(y)$$

$$F(x,y) = 3x^2y^2 - xy^3 + h(x)$$

we conclude that $g(y) = h(x)$

is some constant c.

$$I = \int_S d\omega$$

$$= \int_{\partial S} \omega$$

where $\omega =$
$F(x,y) = 3x^2y^2 - xy^3$

$$= \omega(Q) - \omega(P)$$

$$= F(3,4) - F(1,1)$$

$$= \left(3.9.16 - 3.4^3\right) - (3-1)$$

$$= 236.$$

Continuity

A function $f(x,y)$ is continuous if a small change in input yields only a small change in output.

More formally, $f(x,y)$ is continuous at a point (x_0, y_0) if for any $\varepsilon > 0$ we can find a $\delta > 0$ such that $f(x,y)$ is defined and

$$|f(x,y) - f(x_0,y_0)| < \varepsilon$$

whenever

$|x - x_0| < \delta$ and $|y - y_0| < \delta$.