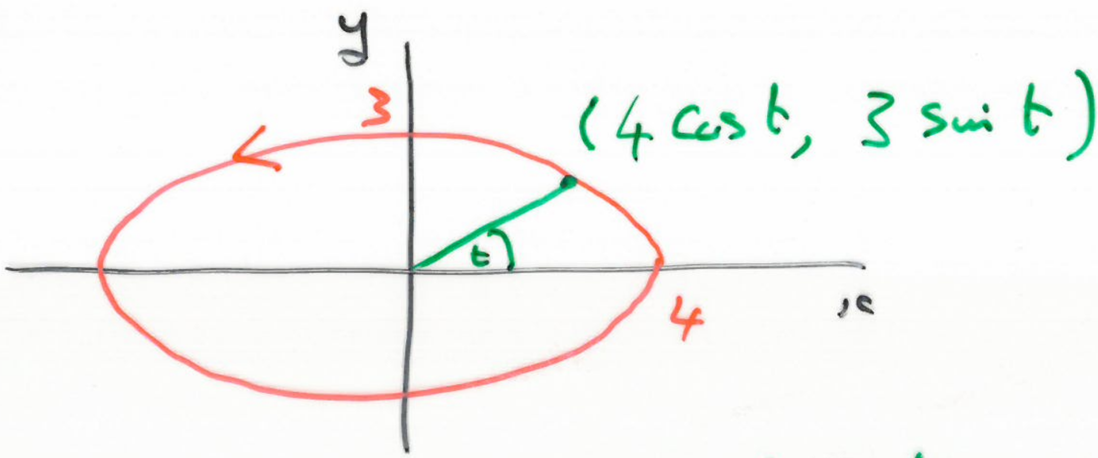
Test: Wednesday 10 October

(Material based on lectures up to 3rd October.) i.e. up to end of §7.)

<u>Sol^n</u>

$$\text{work} = \int_S \omega$$

$$= \int_S (3x - 4y)\,dx + (4x + 2y)\,dy$$



$$x = 4\cos t$$
$$y = 3\sin t$$
$$dx = -4\sin t\,dt$$
$$dy = 3\cos t\,dt$$

work =

$$\int_0^{2\pi} \Big(3\,(4\cos t) - 4\,(3\sin t)\Big)(-4\sin t)\,dt + \Big(4\,(4\cos t) + 2\,(3\sin t)\Big)\,3\cos t\,dt$$

$= \ldots$

$$= \int_0^{2\pi} 48 - 30\,(\sin t)(\cos(t))\;dt$$

$$= 48t - 15\sin^2 t\,\Big|_0^{2\pi}$$

$= 96\pi.$

Stokes' Formula

$$\boxed{\int_{\partial S} \omega = \int_{S} d\omega}$$

when $\omega = f(x_1, x_2, \dots, x_n)$ is a 0-form, and S is a 1-dimensional oriented connected region:

- left-hand side makes sense to us.
- for the right-hand side, we need to give a meaning to $d\omega$.

we need to define the total derivative $d\omega$ of a 0-form. (Also called the exterior derivative.)

Partial Derivatives

Given a 0-form

$$\omega = f(x, y, z)$$

we denote by

$$\frac{\partial f}{\partial x}$$

the 0-form obtained by regarding y and z as constants and differentiating with respect to x, we call $\frac{\partial f}{\partial x}$ the <u>partial derivative</u> of f with respect to x.

Example Consider

$$\omega = f(x,y,z) = \sqrt{1-(x^2+y^2+z^2)}$$

defined on

$$S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 1\}.$$

calculate $\frac{\partial f}{\partial x}$

Sol^n

$$f(x,y,z) = (1-(x^2+y^2+z^2))^{\frac{1}{2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(1-(x^2+y^2+z^2))^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x}{\sqrt{1-(x^2+y^2+z^2)}}.$$

Similarly

$$\frac{\partial f}{\partial y} = \frac{-y}{\sqrt{1-(x^2+y^2+z^2)}}$$

$$\frac{\partial f}{\partial z} = \frac{-z}{\sqrt{1-(x^2+y^2+z^2)}}$$

Notation

we often write

$$f_x$$

in place of

$$\frac{\partial f}{\partial x} .$$

Total derivatives

Given a 0-form

$$\omega = f(x, y, z)$$

we define the 1-form

$$\boxed{d\omega = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz}$$

we call $d\omega$ the total derivative

of the 0-form ω.

Example Find the total derivative of the 0-form

$$\omega = \sqrt{1-(x^2+y^2+z^2)}$$

on $S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 1\}$.

Sol^n

$$d\omega = \frac{-x}{\sqrt{1-(x^2+y^2+z^2)}}\,dx$$

$$- \frac{y}{\sqrt{1-(x^2+y^2+z^2)}}\,dy$$

$$- \frac{z}{\sqrt{1-(x^2+y^2+z^2)}}\,dz.$$