First MA2286 Test: Wed 10 Oct

Example A particle is moved in a constant force field. It takes 3 units of work to move the particle from point (x,y) to point $(x+1,y)$. It takes 4 units of work to move the particle from point (x,y) to point $(x,y+1)$.

We say that work is represented by the 1-form

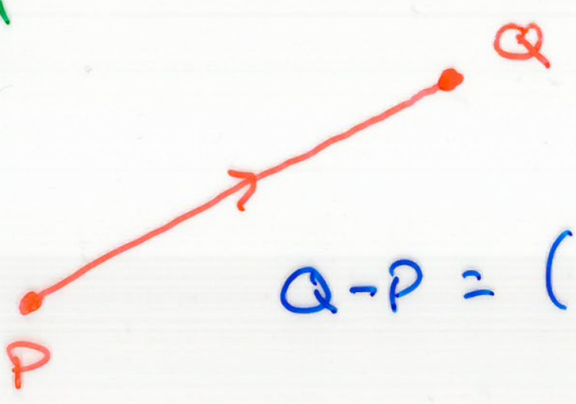
$$\omega = 3\,dx + 4\,dy$$

<u>Example</u> Consider a particle in a constant force field, with work given by the 1-form

$$\omega = 2dx + 3dy + 5dz.$$

Calculate the work done in moving the particle along the straight line segment from point $P = (-1, 3, -5)$ to point $Q = (3, -1, 7)$.

<u>Solⁿ</u>



$$Q - P = (4, -4, 12)$$

$$\text{work} = 2(4) + 3(-4) + 5(12)$$
$$= 56$$

<u>Example</u> An investment portfolio involves two types of assets: type X and type Y.

It costs €3 to acquire one unit of asset X, and -€X to relinquish one unit of asset X. It costs €4 to acquire one unit of asset Y, and -€4 to relinquish one unit of Y.

We say that the marginal costs are represented by the 1-form

$$\omega = 3dx + 4dy$$

Example Find the 1-form

$$\omega = A\,dx + B\,dy + C\,dz$$

describing work in the constant force field, where displacement of a particle from

$(0,0,0)$ to $(4,0,0)$ needs 3 units of work

$(1,-1,0)$ to $(1,1,0)$ " 2 " "

$(0,0,0)$ to $(3,0,2)$ " 5 " "

Sol^n

$$\left.\begin{aligned} 3 &= A.4 \\ 2 &= B.2 \\ 5 &= A.3 + C.2 \end{aligned}\right\} \quad \begin{aligned} A &= \frac{3}{4} \\ B &= 1 \\ C &= \frac{11}{8} \end{aligned}$$

$$5 = 3\left(\frac{3}{4}\right) + 2C$$

$$5 = \frac{9}{4} + 2C$$

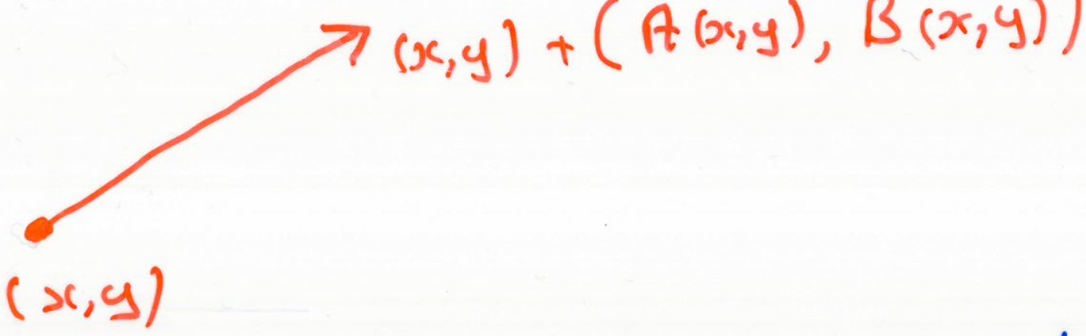
$$\frac{20-9}{4} = 2C$$

We can think of a 1-form

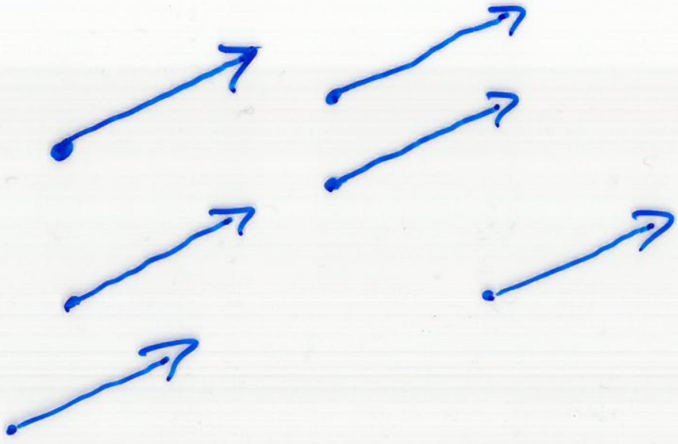
$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

as a collection of arrows in space (= plane for two variables).

For each point (x,y) in the plane we have an arrow



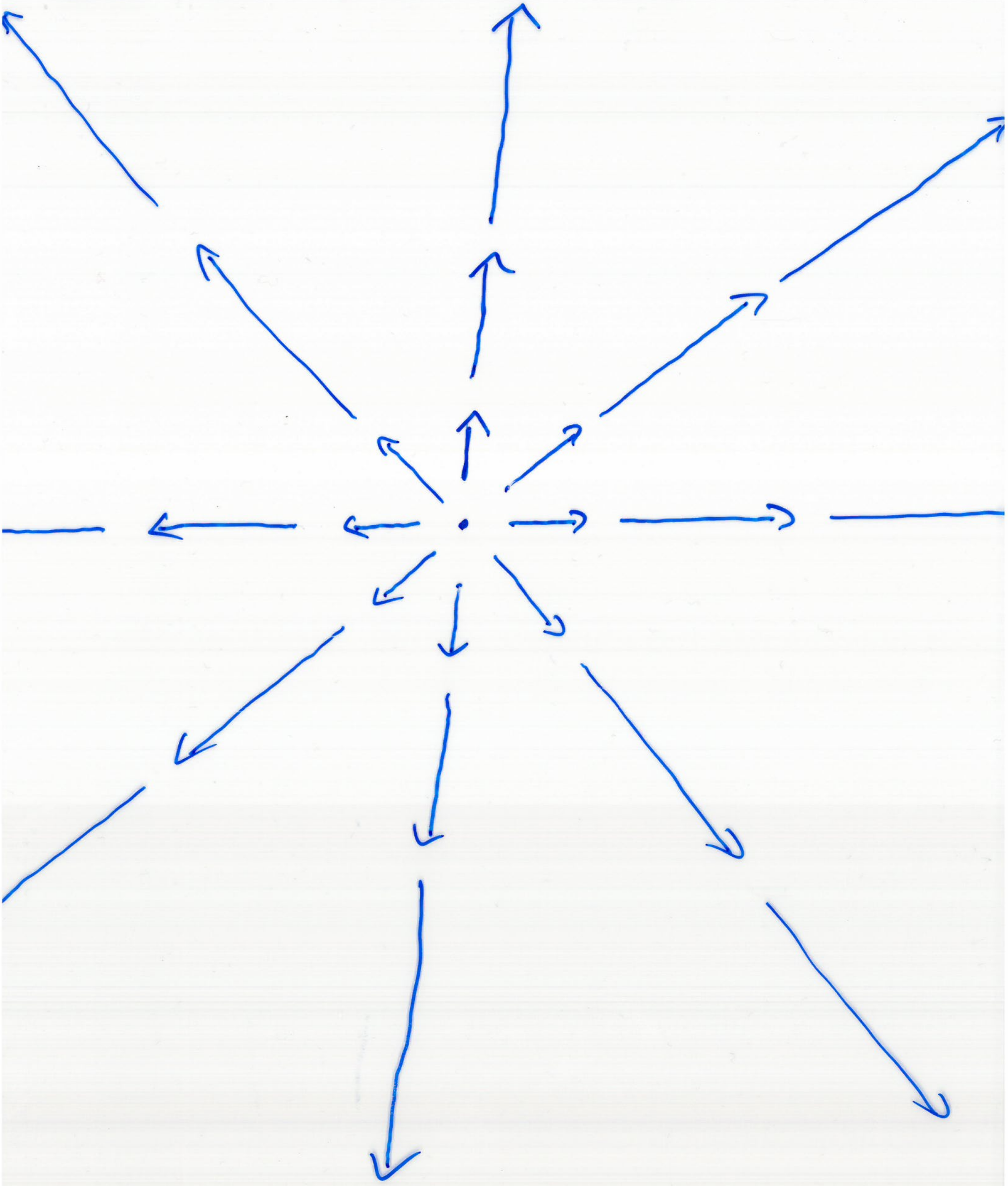
<u>Example</u> The 1-form $\omega = 2dx + dy$ can be pictured as



Example The 1-form

$$\omega = x\,dx + y\,dy$$

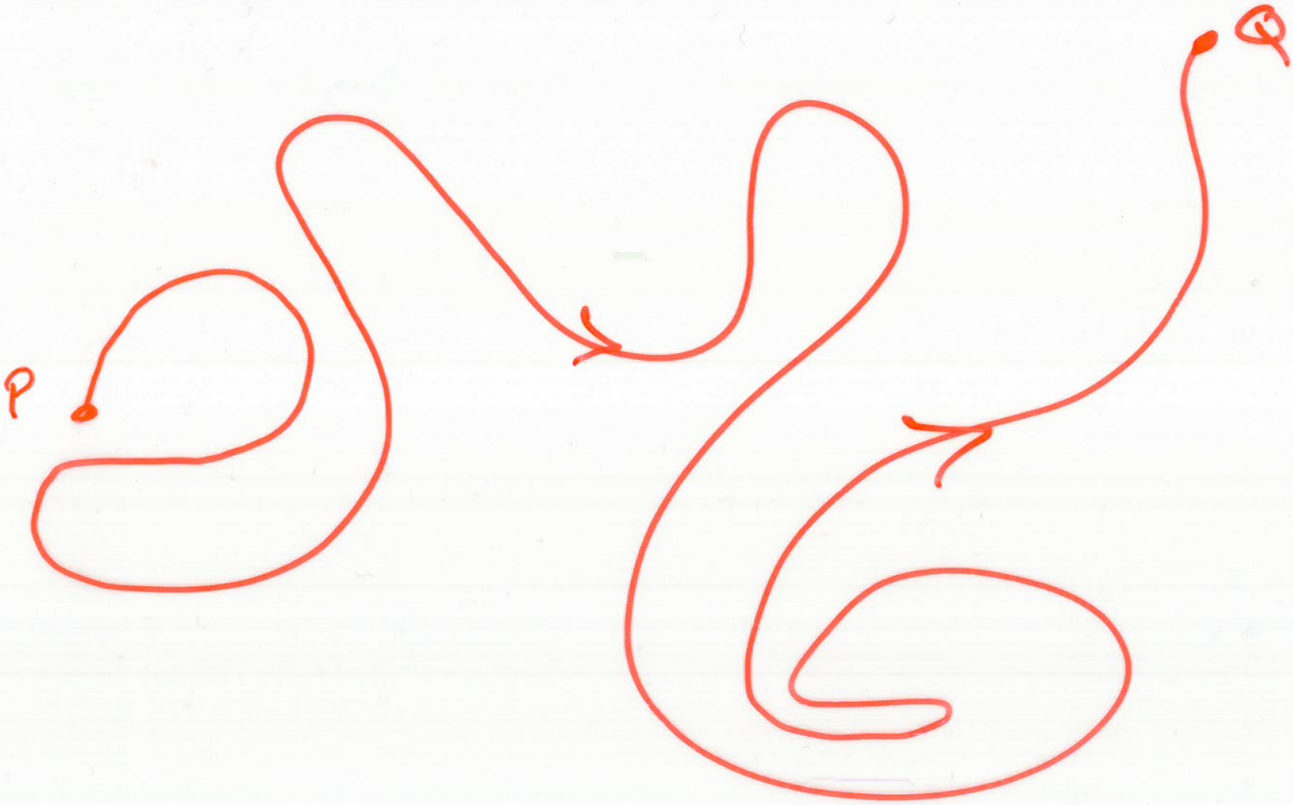
can be pictured as



Integration of 1-forms

Let $\omega = A(x,y)\,dx + B(x,y)\,dy$ be a 1-form.

Let $S \subseteq \mathbb{R}^2$ be a 1-dimensional, oriented, connected subset.



__Informally__: if we think of ω as a "work 1-form" then

$$\int_S A(x,y)\,dx + B(x,y)\,dy$$

is the total work done in moving the particle from P to Q along S.