Continued from last lecture:

$$\cos(x) = \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$$

$$= \frac{1}{1+u^2} - \frac{u^2}{1+u^2}$$

$$= \frac{1-u^2}{1+u^2}$$

$$dx = 2\cos^2\left(\frac{x}{2}\right) du$$

So,

$$w = \int \frac{1}{5+3\cos(x)} \, dx$$

$$= \int \left(\frac{1}{5+3\left(\frac{1-u^2}{1+u^2}\right)}\right) \frac{2}{1+u^2} \, du$$

$$= \ldots$$

$$= \int \frac{1}{4+u^2} \, du$$

From the log book:

$$w = \frac{1}{2} \tan^{-1} \frac{u}{2} + C$$

$$\boxed{w = \frac{1}{2} \tan^{-1} \left( \frac{1}{2} \tan\left(\frac{x}{2}\right) \right) + c}$$

# Differential 0-forms on n-dimensional space

A **differential 0-form** on 2-dimensional space is a real valued function

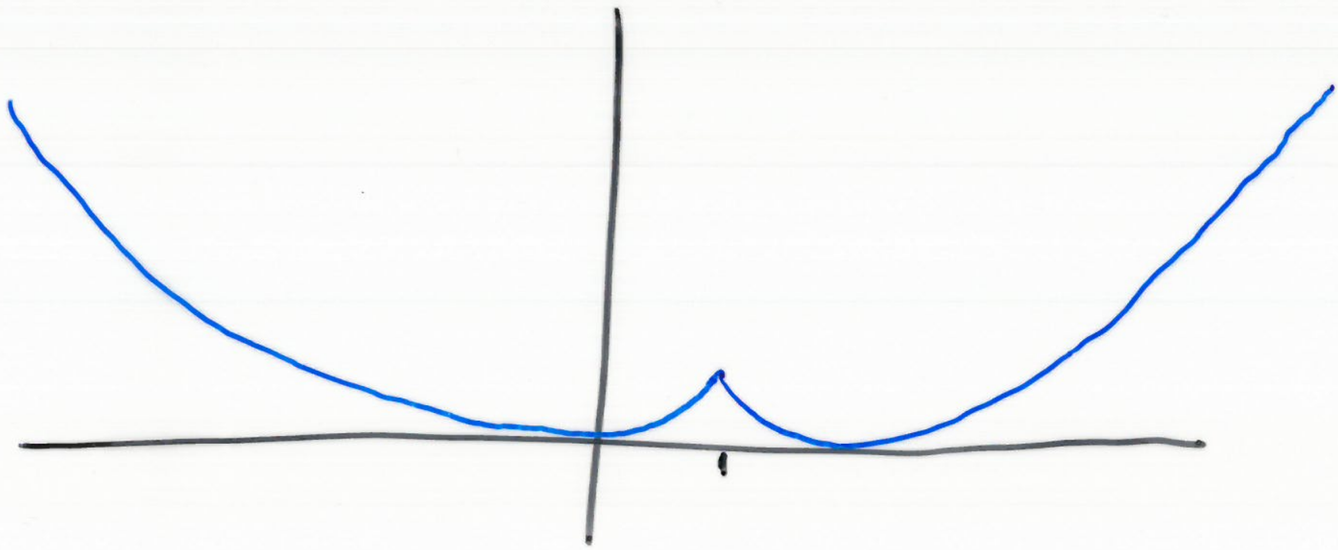
$$\omega = f(x,y)$$

which is "differentiable". To explain this term, recall:

**informally**: A function $f(x)$ is differentiable at a point $x$ if the curve $y = f(x)$ has a well-defined (= unique) tangent line at $x$.

## Example

$$y = \begin{cases} x^2 & x \le 1 \\ (x-2)^2 & x > 1 \end{cases}$$



This is **not** differentiable at $x = 1$.

**Informally** A function $f(x,y)$ is differentiable at a point $(x,y)$ if the surface

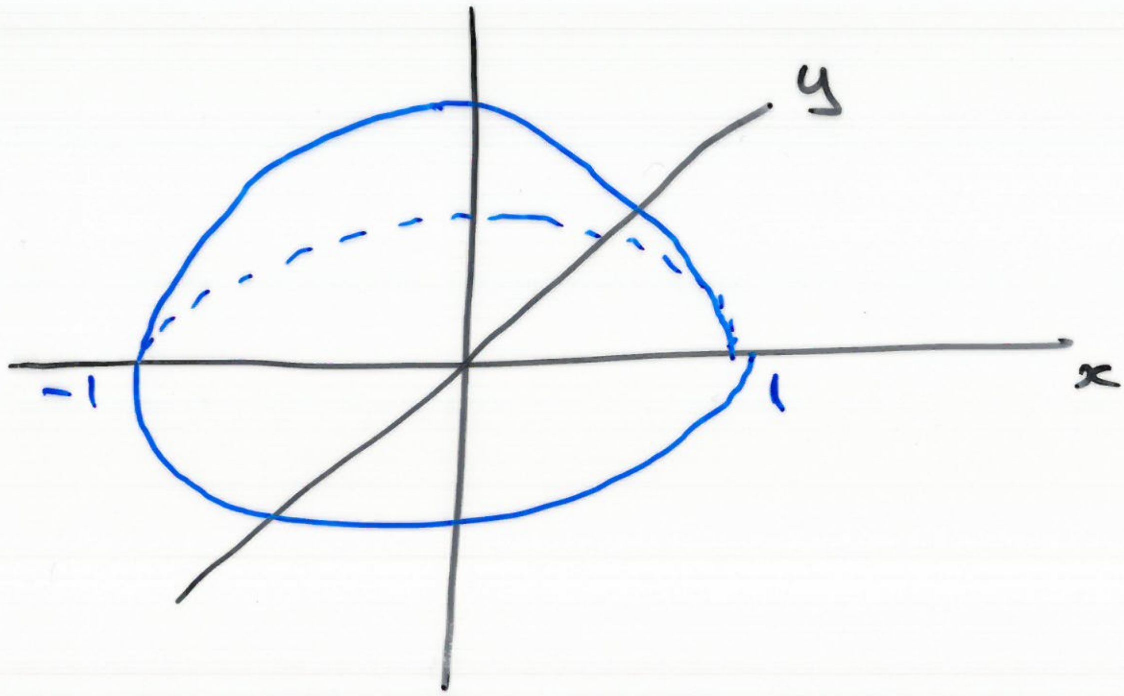
$$z = f(x,y)$$

has a well-defined tangent plane.

<u>Example</u> $z = \sqrt{1-x^2-y^2}$ is

defined $x^2+y^2 \leq 1$, and

describes the surface




for any point $(x,y)$ in

$S = \{ (x,y) : x^2+y^2 \leq 1 \}$

the surface has a well-defined

tangent plane.

So

$$w = \sqrt{1 - x^2 - y^2}$$

is a differential 0-form on $S$.

Let's skip the formal definition of differentiability.

## Differential 1-forms on n-dimensional space.

A differential 1-form on a 2-dimensional region $S$ is a function

$$\omega = A(x,y)\, h_1 + B(x,y)\, h_2$$

that inputs a vector $(x,y)$ and a vector $(h_1, h_2)$ and returns a real number.

Here $A(x,y)$ and $B(x,y)$

must be differentiable real-valued functions.

<u>Example</u> Evaluate the 1-form

$$\omega = (x^2+y^2)\, h_1 + 2xy\, h_2$$

at $(x,y) = (2,4)$ and $(h_1, h_2) = (\frac{1}{4}, \frac{1}{4})$.

<u>Sol$^n$</u> 9

<u>Notation</u> We usually denote

$$\omega = A(x,y)\, h_1 + B(x,y)\, h_2$$

by

$$\omega = A(x,y)\, dx + B(x,y)\, dy .$$

<u>Example</u> Evaluate the 1-form

$$\omega = (x^2+y^2)\,dx + 2xy\,dy$$

at $(x,y) = (2,4)$, $(dx,dy) = (\frac{1}{4}, \frac{1}{4})$.

<u>Sol<sup></sup>n</u> 9.