MA2286 Differential Forms (2018-19)

(= Calculus)

Aim: Explain and apply the generalized Stokes formula:

$$\int_{\partial S} \omega = \int_S \partial \omega$$

where

- ω is a differential p-form in n variables

- S is a nice region in $\mathbb{R}^n$.

- ∂S is the boundary of S

- $\int$ is an integral.

Differential 0-forms in 1 variable

$(p=0, \; n=1)$

A differential 0-form in 1 variable is just a differentiable real valued function

$$\omega = f(x) .$$

Examples

$$\omega = 3x - 4$$
$$\omega = 3x^2 + 4$$
$$\omega = x \sin(x)$$

are differential 0-forms.

usually a diff. 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_k, b_k]$$

we only require ω to be differentiable on S.

Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 1000]$$

Clearly ω is not a differential 0-form on

$$S = [-1, 1].$$

Terminology

We'll say 0-form instead of differential 0-form.

for $a < b \in \mathbb{R}$ we write

$$[a, b] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

and we picture this as

The arrow is an <u>orientation</u> that specifies the direction of travel from a to b.

For $a < b \in \mathbb{R}$ we write

$$[b, a] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

and picture this as

we say that $[a, b]$ and $[b, a]$ are <u>oriented intervals</u>.

<u>Example</u> $S = [2, 1] \cup [3, 4] \cup [6, 5]$

The boundary of the oriented interval $S = [a, b]$ is the set

$$\partial S = \{ a, b \}$$

The set consisting of two points, the initial point a and final point b.

Example

$$S = [2,1] \cup [3,4] \cup [6,5]$$

$$\partial S = \{1, 2, 3, 4, 5, 6\}.$$

Terminology We'll say that $S = [a,b]$ is 1-dimensional, and that the boundary ∂S is 0-dimensional.

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = F(b) - F(a)$$

example integrate the differential 0-form

$$\omega = 3x^2 + 4$$

over the boundary of the oriented interval

$$S = [2, 1].$$

Soln

$$\int_{\partial S} \omega = \omega(1) - \omega(2)$$

$$= 7 - 16 = -9.$$