

MA2286 Differential Forms (2018-19)  
(= Calculus)

Aim: Explain and apply the  
generalized Stokes formula:

$$\int_{\partial S} \omega = \int_S d\omega$$

where

- $\omega$  is a differential  $p$ -form  
in  $n$  variables
- $S$  is a nice region in  $\mathbb{R}^n$ .
- $\partial S$  is the boundary of  $S$
- $\int$  is an integral.

# Differential 0-forms in 1 variable

$$(p=0, n=1)$$

2

A differential 0-form in 1 variable is just a differentiable real valued function

$$\omega = f(x)$$

Example

$$\omega = 3x - 4$$

$$\omega = 3x^2 + 4$$

$$\omega = x \sin(x)$$

are differential 0-forms.

usually a diff. 0-form is given in the context of some closed interval

$$S = [a, b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1, b_1] \cup [a_2, b_2] \cup \dots \cup [a_k, b_k]$$

We only require  $\omega$  to be ③  
differentiable on  $S$ .

### Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 1000]$$

Clearly  $\omega$  is not a differential  
0-form on

$$S = [-1, 1].$$

### Terminology

Will say 0-form instead of  
differential 0-form.

For  $a < b \in \mathbb{R}$  we write

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and we picture this as

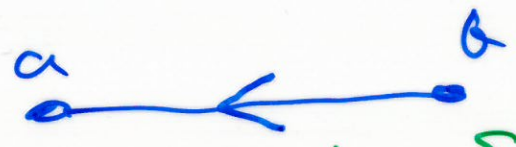


The arrow is an orientation that specifies the direction of travel from  $a$  to  $b$ .

For  $a < b \in \mathbb{R}$  we write

$$[b, a] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

and picture this as



We say that  $[a, b]$  and  $[b, a]$  are oriented intervals.

Example  $S = [2, 1] \cup [3, 4] \cup [6, 5]$



The boundary of the oriented interval  $S = [a, b]$  is the set

$$\partial S = \{a, b\}$$

The set consisting of two points, the initial point  $a$  and final point  $b$ .

(5)

Example

$$S = [2, 1] \cup [3, 4] \cup [6, 5]$$

$$\partial S = \{1, 2, 3, 4, 5, 6\}.$$

Terminology We'll say

that  $S = [a, b]$  is

1-dimensional, and that the  
boundary  $\partial S$  is 0-dimensional.

Definition Given a 0-form

6

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = F(b) - F(a)$$

example integrate the differential 0-form

$$\omega = 3x^2 + 4$$

over the boundary of the oriented interval

$$S = [2, 1].$$

Soln

$$\begin{aligned} \int_{\partial S} \omega &= \omega(1) - \omega(2) \\ &= 7 - 16 = -9. \end{aligned}$$