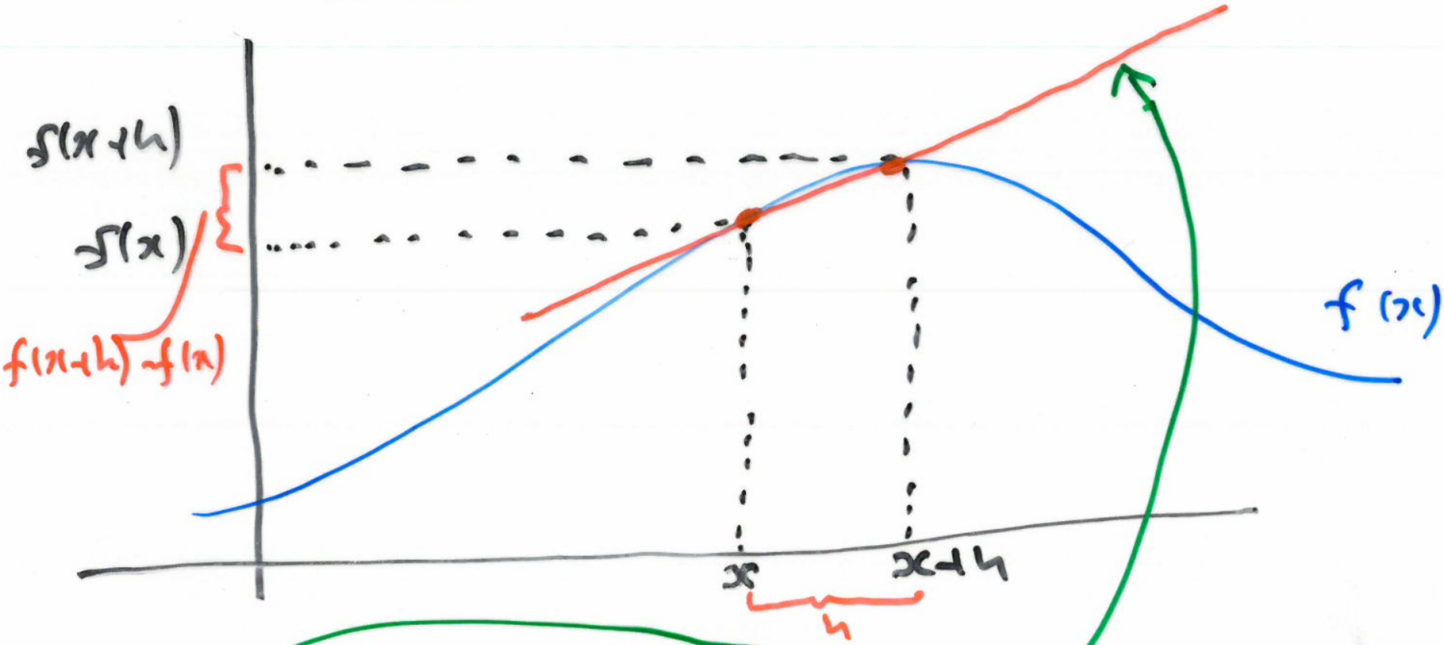
Aisling.Mccluskey@nuigalway.ie

Irish workshop

Derivative



$$\frac{f(x+h)-f(x)}{h}$$ = slope of red line.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ = slope of tangent to curve $y=f(x)$ at x.

Definition

Remember

$$\frac{d}{dx} f(x) = f'(x)$$

Rules of differentiation

$$\boxed{\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)}$$

SUM RULE

Proof

LHS =

$$\lim_{h\to 0} \frac{f(x+h)+g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right) + \lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right)$$

$$= \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

Example

$$\frac{d}{dx}\left(x^{\frac{3}{2}} + \sin(x)\right)$$

$$= \frac{d}{dx}x^{\frac{3}{2}} + \frac{d}{dx}\sin(x)$$

$$= \frac{3}{2}x^{\frac{1}{2}} + \cos(x)$$

$$\boxed{\frac{d}{dx}\left(k f(x)\right) = k\frac{d}{dx}f(x)}$$

Scalar Product Rule

(where k is any constant)

Example

$$\frac{d}{dx}\left(3e^{x}\right) = 3\frac{d}{dx}e^{x} = 3e^{x}.$$

$$\boxed{\begin{aligned}&\frac{d}{dx}\left(f(x)\,g(x)\right)\\&=\left[\frac{d}{dx}f(x)\right]g(x)+f(x)\left[\frac{d}{dx}g(x)\right]\end{aligned}}$$

PRODUCT RULE

Example

$$\frac{d}{dx}\left(x^2 \sin x\right)$$

$$=\left[\frac{d}{dx}x^2\right]\sin(x)+x^2\left[\frac{d}{dx}\sin(x)\right]$$

$$= 2x\sin x + x^2\cos(x).$$

Example

$$y=(x^2+1)(x^3+2)$$

$$\frac{dy}{dx}=\left[\frac{d}{dx}(x^2+1)\right](x^3+2)+(x^2+1)\left[\frac{d}{dx}(x^3+2)\right]$$

$$= 2x(x^3+2) + (x^2+1)(3x^2)$$

$$= 5x^4 + 3x^2 + 4x$$

Chain Rule

Given functions

$f(x)$ and $g(x)$

we can consider the

composite function

$$y = g(f(x))$$

$$\boxed{\frac{dy}{dx} = g'(f(x))\, f'(x)}$$

CHAIN RULE

Example $y = \sin(x^2)$

$f(x) = x^2, \quad g(x) = \sin(x)$

Sol^n

$$\frac{dy}{dx} = \cos(x^2)\, 2x$$

$$= 2x \cos(x^2)$$

Example

$$y = (x^2 - x + 1)^7$$

$$\frac{dy}{dx} = 7(x^2 - x + 1)^6 (2x - 1)$$

Example

$$y = \sqrt{x^2 + 1}.$$

$$y = (x^2 + 1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} (2x)$$

$$= \frac{x}{\sqrt{x^2 + 1}}.$$

Quotient Rule

$$y = \frac{f(x)}{g(x)}$$

$$y = f(x)\left(g(x)\right)^{-1}$$

$$\frac{dy}{dx} = f'(x)\left(g(x)\right)^{-1} + f(x)\left[\frac{d}{dx}\left(g(x)\right)^{-1}\right]$$

$$\frac{dy}{dx} = f'(x)\left(g(x)\right)^{-1} - f(x)\left(g(x)\right)^{-2} g'(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{g(x)} - \frac{f(x)\,g'(x)}{\left(g(x)\right)^{2}}$$

$$\boxed{\frac{dy}{dx} = \frac{f'(x)g(x) - f(x)\,g'(x)}{g(x)^{2}}}$$

QUOTIENT RULE,