Limits at infinity

Defn

$$\lim_{x \to \infty} f(x) = \lim_{x \to 0^+} f\left(\frac{1}{x}\right)$$

Defn

$$\lim_{x \to -\infty} f(x) = \lim_{x \to 0^-} f\left(\frac{1}{x}\right)$$

Example Evaluate

$$l = \lim_{x \to \infty} \frac{1}{\sqrt{x^2+2x} - x}$$

Sol^n

$$l = \lim_{x \to \infty} \frac{1}{\sqrt{x^2+2x} - x} \cdot \frac{(\sqrt{x^2+2x} + x)}{(\sqrt{x^2+2x} + x)}$$

$$l = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x^2+2x-x^2}$$

$$l = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{2x}$$

$$l = \lim_{x \to \infty} \left(\frac{\sqrt{x^2+2x}}{2x} + \frac{1}{2} \right)$$

$$l = \lim_{x \to \infty} \left(\sqrt{\frac{x^2+2x}{4x^2}} + \frac{1}{2} \right)$$

$$l = \lim_{x \to \infty} \left(\sqrt{\frac{1}{4} + \frac{1}{2x}} + \frac{1}{2} \right)$$

$$l = \frac{1}{2} + \frac{1}{2}$$

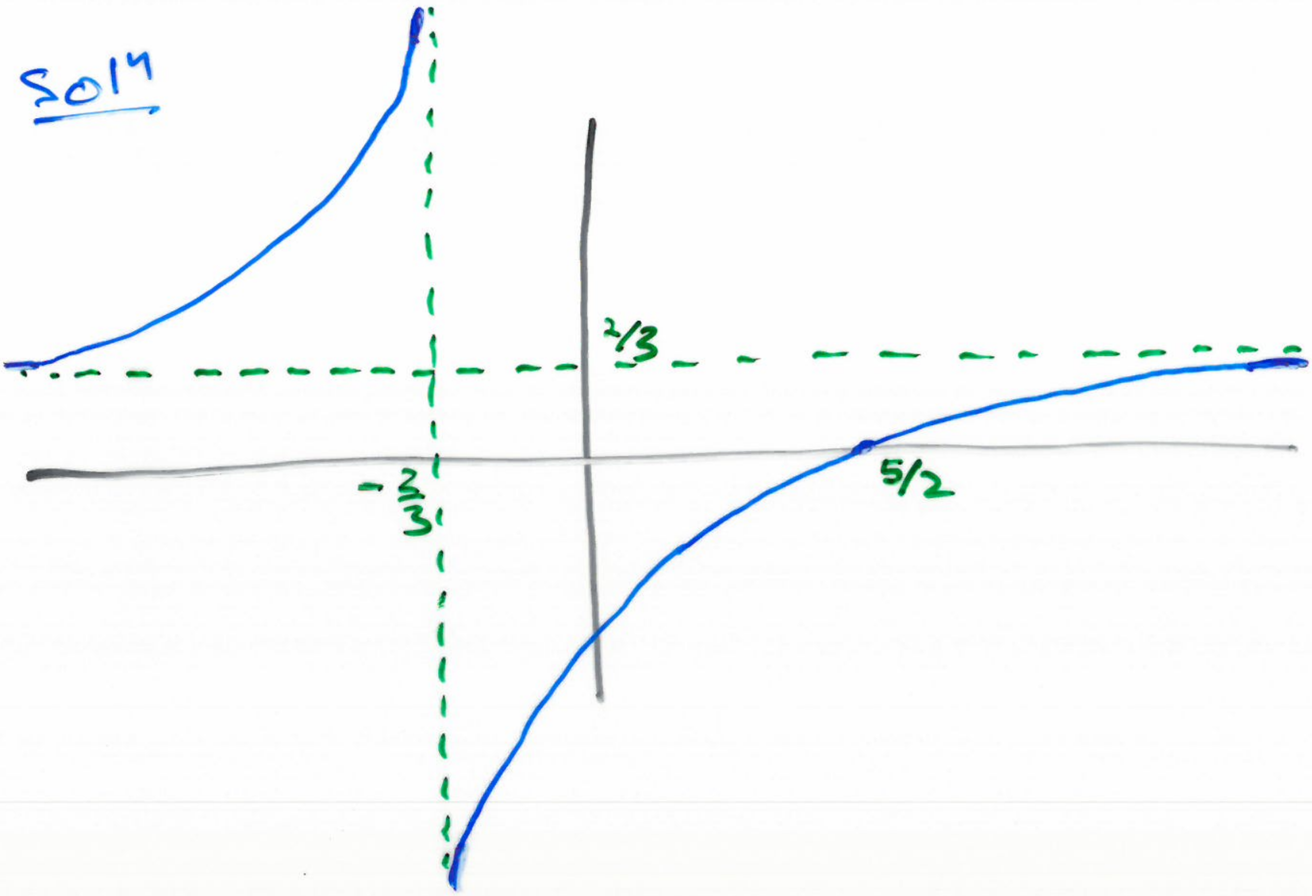
$$\underline{\underline{l = 1}}$$

Example what are the horizontal and vertical asymptotes of

$$y = \frac{2x-5}{3x+2} \quad ?$$

sketch the graph of y.

Solⁿ



$$\lim_{x \to \infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

So the limits at infinity correspond to horizontal asymptotes.

Note: this function y has domain

$$\mathrm{Domain}(y) = \mathbb{R} \setminus \{-\tfrac{2}{3}\},$$

Note: $y = f(x)$ is a function

$$f : \mathbb{R} \setminus \{-\tfrac{2}{3}\} \longrightarrow \mathbb{R}$$

and it <u>is</u> continuous.

Topic II

Rates of change

Differentiation

Given a function $f(x)$ we define the **derivative** to be a function $f'(x)$ defined as

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Example: Find the derivative $f'(x)$ of the function $f(x) = x^2$.

Sol^n

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x^2 + 2hx + h^2) - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h$$

$$\underline{\underline{f'(x) = 2x}}\ .$$

So for $f(x) = x^2$ we have

$f'(x) = 2x$.

Derivatives of some basic functions

For $y = f(x)$ we often write

$$\frac{dy}{dx}$$

instead of $f'(x)$.

- $\frac{d}{dx} x^n = n x^{n-1}$ for any $n \neq 0$

- $\frac{d}{dx} \sin x = \cos(x)$

- $\frac{d}{dx} \cos x = -\sin(x)$

- $e = 2.71\ldots\ldots$

  $\frac{d}{dx} e^x = e^x$

- etc.