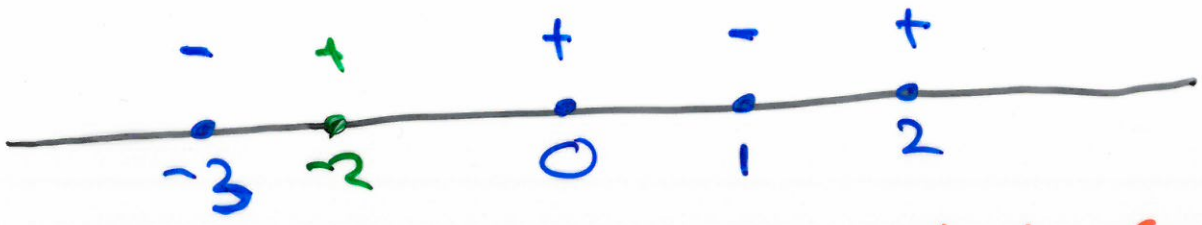
Example Show that

$$x^3 - 4x + 1 = 0$$

has three real solutions, and find approximations to them.

Soln

$$f(x) = x^3 - 4x + 1$$



$f(0) > 0$

$f(1) < 0$

$f(2) > 0$

$f(-3) < 0$

$f(-2) > 0$

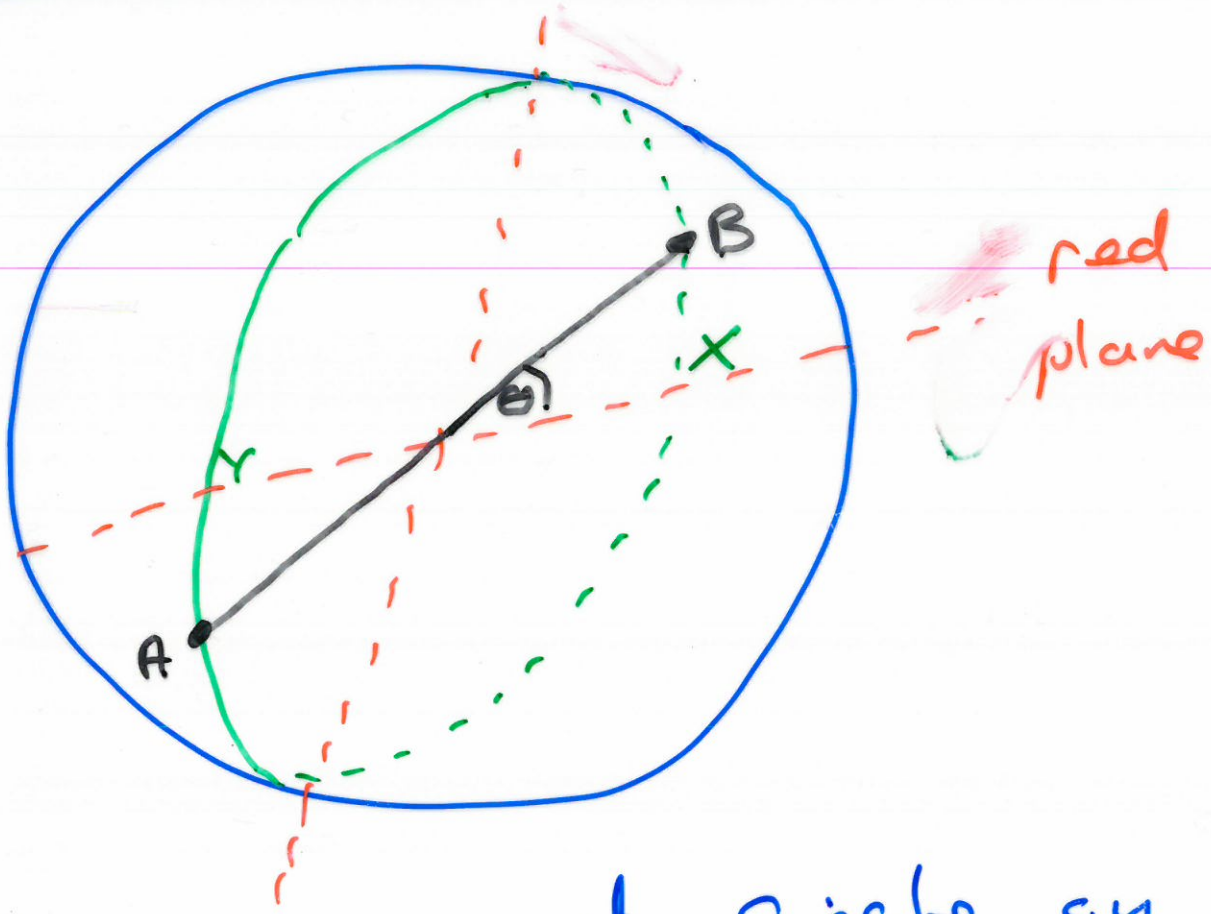
IVT says that $f(x) = 0$ has three solutions:

$c_1 \in [-3, -2]$

$c_2 \in [0, 1]$

$c_3 \in [1, 2]$.

Application of IVT



Take any great circle on the Earth.

Fact: There exist two opposite points on your great circle with equal air pressure.

Explanation of this fact

Consider

$f(\theta)$ = air pressure at A
− air pressure at B.

Note: $f(\theta)$ is a continuous function of θ.

I want to prove that for some value of the angle θ pressure at A = pressure at B.

i.e. I want to prove that

$f(\theta) = 0$

for some angle $\theta \in [0, \pi]$.

If $f(0) = 0$, or if $f(\pi) = 0$ then we are done!

Suppose then that

$f(0) \neq 0$ and $f(\pi) \neq 0$.

Note: $f(0) f(\pi) < 0$

$f(0)$ = air pressure at x
− air pressure at Y

$f(\pi)$ = air pressure a Y
− air pressure at x.

So $f(0) = -f(\pi)$.

So IVT says there is some $\theta \in [0, \pi]$ such that $f(\theta) = 0$.

QED